ECE XX: Computational Power Flow Analysis
Course Portfolio

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Contents

1 Course Description

In this course, we will embark on a journey to unravel the *power flow equations*—the fundamental laws that govern our electric power grid. Despite more than a century of study, efficiently solving these equations remains a captivating challenge, with the total number of solutions still an unsolved mystery. Together, we will dive deep into the mathematical structure of these equations, explore their various representations, and learn how to harness scientific computing techniques to find practical solutions that power our world.

By the end of this class, you will be well-prepared to *computationally model power flow* in electrical networks and integrate these models into engineering decision-making processes like optimization, control, and statistical analysis. These skills are crucial for addressing real-world challenges such as integrating renewable energy sources, improving the reliability of the power grid, and optimizing energy distribution. Moreover, we will explore surprising connections between power systems and broader mathematical fields like graph theory and optimization, preparing you for advanced studies in smart grid technologies, sustainable energy systems, and interdisciplinary research beyond the classroom.

Figure 1: The electric power grid is the most intricate and complex machine ever built by humans. It is a vast network *of interconnected components, with incessantly fluctuating demand. The grid's exigency to balance generation with consumption across a large geographical area makes it a highly complex system to manage and maintain.*
Source: Effects of the 2021 Freeze on Houston, Texas, in the ERCOT Power Grid: NASA Earth Observatory/NASA Earth Observatory

2 Reflection

2.1 Feedback and major changes to course design

The review process for my course design has been an immensely valuable experience. As a whole, working through the process of CETL 8717 has given me a great number of both constructive feedback and actionable suggestions for improvement. Feedback from my Learning Community during our regular in-person meetings has helped refine key components of the course. My Learning Community and I are planning to meet again this week, so I will likely update this submission with more detailed information from our peer review sessions.

In particular, Dr. Nolen's detailed feedback on five of the seven assignments was especially insightful, and gave me fuel for major revisions. For instance, based on Dr. Nolen's suggestion, I am developing a new learning objective that addresses the human dimension of learning from Fink's Taxonomy; I agree it is essential to emphasize the broader personal impact of the course content. Additionally, I am adjusting my exam wrapper to focus more on students' reflections about their strengths and weaknesses during the exam process rather than their preparation strategies. Similarly, I'm revising the constraints for the take-home format to make them easier to enforce. Finally, I am re-evaluating my lesson plans to address concerns about pacing, ensuring time is allocated for clarifications and student engagement. I believe these changes will significantly improve the course, making it more efficient, as well as supportive and inclusive for students.

2.2 Reflection on course design process

Over the course of this semester, my approach to course design has evolved significantly, in both theory and practice. My role as an instructor of record for ECE 2020 provided new firsthand experience, while CETL 8717 educated me on evidence-based design principles like Backwards Design and Universal Design for Learning (UDL). These frameworks are helping me structure my courses with clearly defined learning objectives, *authentic* assessments, and inclusive lesson plans.

As I design courses in the future, perhaps my primary focus will be on leveraging peer-reviewed Taxonomies like Bloom's and Fink's. I've gained significant insight from CETL 8717 in how to use these taxonomies as a lens to scaffold objectives across cognitive levels, ensuring both depth and breadth in learning. This process has significantly help me better shape my course into a cohesive and intentional learning experience that prioritizes student engagement and success. Moving forward, I will use these principles as a foundation for my career; I will have a sharp focus on inclusivity and adaptability in my course design to meet diverse student needs. This semester, as an instructor of record, I have emphasized communication, reflection, and continual improvement, and I aim to continue to create courses by applying these techniques. My hope is that these methods can help inspire curiosity and growth in Electrical Engineering students.

2.3 Note on revised content

The content throughout this portfolio has been revised according to the great and insightful feedback I have received throughout the semester. Several places where specific revisions were requested have indications in **purple**; however, I want to emphasize that the these markings are *not* all-encompassing of the revisions.

Thanks for a great semester!

3 Student learning objectives (SLOs)

Below, we develop several student learning objectives (SLOs), where each objective is broken up into component skills (CS). Each SLO and CS aims to be appropriately distributed across Bloom's and Fink's taxonomies. Moreover, each of these learning objectives and component skills strives to be specific, action-oriented, and measurable.

Upon successful completion of this course, students should be able to:

- SLO 1: *Construct* matrix-based network models for electric power systems.
  - CS 1.1: Combine linear algebra and circuit analysis principles to *construct* matrix-based models for the power flow equations.
  - CS 1.2: Identify the difference between bus injection and branch flow models of power flow equations and correctly transform between them.
  - CS 1.3: Assess the context of a power flow problem, such as network size, margins of error, and population demographics, when choosing a representation of the power flow equations.
- SLO 2: *Recognize* the appropriateness of using different representations of the power flow equations to perform a power flow analysis.
  - CS 2.1: Describe strengths and weaknesses of relaxations and approximations of the power flow equations.
  - CS 2.2: Evaluate whether a *distribution* (small-scale) or *transmission* (large-scale) network model is appropriate for a given problem.
  - CS 2.3: Contrast the difference between a relaxation (i.e., allowing flexibility) and an approximation (i.e., using different, approximate forms) of the power flow equations.
- SLO 3: *Develop* optimization and control programs whose solutions are ensured to satisfy the power flow equations.
  - CS 3.1: Combine the power flow equations with common optimization goals, such as minimizing operating cost or voltage instability.
  - CS 3.2: Build engineering constraint criteria that enforce the satisfaction of the power flow equations in a solution to an optimization problem.
  - CS 3.3: Analyze the computational costs and efficiency of optimization and control programs when supplied with different representations of the power flow equations.
- SLO 4: *Predict* future grid operating conditions by *constructing* and *testing* statistical forms of the power flow equations.
  - CS 4.1: Estimate a power network's operating state from measurement data streams.
  - CS 4.2: Predict and identify unfavorable grid operating conditions when uncertainty or randomness is specified in a network.
  - CS 4.3: Appraise the capacity of a given power network to host renewable energy, and propose network upgrades and modifications to increase this capacity.
  - CS 4.4: Reconstruct the power flow equations from measurement data, without an underlying network model.
- SLO 5: *Summarize* the ethical considerations of different computational and statistical models for the power flow equations.

- CS 5.1: Identify potential ethical shortcomings in power flow analyses, such as fairness in electricity pricing and socioeconomic disparities in access to renewable energy.
- CS 5.2: Explain the impact of how we design our computational power flow models on these ethical considerations.
- CS 5.3: Assess the societal impact of a computational power flow model; *defend* its potential ethical strengths and critique its potential ethical weaknesses.

- SLO 6: *Demonstrate* how to manage large data sets of measurements collected from real-world electric power systems.
  - CS 6.1: Visualize power system measurements using plotting and illustration software, e.g., PowerModels.jl, NetworkX, Matplotlib, and Makie.
  - CS 6.2: Implement circuit calculation functions and manipulate power system datasets using numerical analysis software such as Julia, numpy, or MATLAB.
- SLO 7: *Design* a computational tool that solves optimal power flow problems.
  - CS 7.1: Implement the optimal power flow problem in the Julia or python programming language with an optimization engine.
  - CS 7.2: Test different representations of the power flow equations in this tool
  - CS 7.3: Contrast the effects of different representations on performance, accuracy, and reproducibility.
- **(New) SLO 8:** *Recognize* the the societal impacts of pricing mechanisms in electricity markets.
  - CS 8.1: Explain the use of the *energy burden* metric as a tool for analyzing the fairness of electricity prices.
  - CS 8.2: Assess locational marginal prices (LMPs) using the dual optimal solution to an optimal power flow (OPF) problem.
  - CS 8.3: Analyze disparities in electricity pricing across human communities and demographic groups using locational marginal prices.
  - CS 8.4: Assess the impact of increasingly extreme weather events, such as the 2021 Texas Freeze shown in Fig. 1, on electricity prices, and the disparate impacts these can have on communities.
  - CS 8.5: Appraise the respective merits, risks, and trade-offs of regulated and deregulated electricity markets, particularly in regard to blackout risk.

Learning Objective	Low/No Stakes Assessment 1	Low/No Stakes Assessment 2	Low/No Stakes Assessment 3	Major Assessment
At the end of this course, you should be able to know/do…	***Before Instruction***: Students answer a few questions or complete a task assessing their conceptual understanding about the material.	***During Instruction***: Students answer questions/complete a task relevant to the day's topic.	***Following Instruction***: Students complete an assignment practicing the topic of the class.	How will the students demonstrate mastery of the learning objective?
Construct matrix-based network models for electric power systems.	Narrative discussion in class about how to use Ohm's Law to represent flows in a network. Informally introduce nodal admittance (graph Laplacian) matrix for solving linear graph systems.	Homework (graded for correctness solely for feedback, with unlimited revision opportunities). Project-style problem sets in pairs with peer review. Build up to graph-theoretic interpretation of the power flow equations.	In-class quizzes, Q/A, discussion, reflection (e.g. why is this important, where have I seen this before?)	Midterm exam (with exam wrapper), see Problem 1 of sample exam for example.
Recognize the appropriateness of using different representations of the power flow equations to perform a power flow analysis.	Informal history of how the power flow equations were discovered, classical and contemporary history of relaxations. Informally assess understanding of branch flow/bus injection models by motivating discussion with Europe's single-phase balanced distribution grid vs. North America	Homework (graded for correctness solely for feedback, with unlimited revision opportunities). Project-style problem sets in pairs with peer review. Derive LinDistFlow model from scratch using the power flow Jacobian Matrix.	In-class quizzes Q/A discussion. Self reflection opportunity to identify potential pitfalls in understanding the differences between relaxations and approximations.	Midterm exam (with exam wrapper), see Problem 2 of sample exam for example.
Develop optimization and control programs whose solutions are ensured to satisfy the power flow equations.	Class demonstration with real-world case studies, explaining different optimization methods. Assess informal understanding of linear vs. non-linear optimization.	Homework where students implement optimization programs on small-scale power grids. Metacognitive exercise where students are asked to reflect on different ways to implement these constraints (data-driven vs. relaxation vs. approximation).	Group discussions and feedback sessions on optimization techniques. Self-reflection opportunity to assess the weaknesses of approximations in optimization models.	Midterm exam (with exam wrapper), see Problem 4 for example. Could potentially be tangentially explored in course project depending on topical selection of students.
Predict future grid operating conditions by constructing and testing statistical forms of the power flow equations.	Narrative discussion in class introducing statistical methods for forecasting grid behavior. Assess conceptual through in-class questioning. Group understanding discussion on the appropriateness of learning techniques in power systems.	Homework on creating simple statistical models based on historical grid data. Metacognitive exercise where students are asked to reflect on the pitfalls of deep learning methods in this setting, and the benefits of structured physics-informed learning techniques.	Group project: Construct and present a statistical model predicting grid performance. Gamified in-class "slot-machine" discussion of random power generation from solar panels. Connect randomized analysis to solar weather patterns, extreme events, wildfire risk, etc.	Midterm exam, see Problem 3 and Problem 5 of sample exam for example. Anticipated to be the most likely course project topic to be selected by students.
Summarize the ethical considerations of different computational and statistical models for the power flow equations.	Discussion of the Energy Burden metric. Class discussion on ethical issues in power grid modeling, focusing on fairness and transparency. Informal assessment of students understanding of energy cost disparities across communities, renewable energy access disparities, and privacy	Homework assignment exploring case studies on ethical concerns (e.g., data privacy, accessibility). Assignment develops the Locational Marginal Burden matrix for computing the change in energy burden faced by communities incurred by changes in	Energy burden visualization and discussion. Group debate on ethical trade-offs in different models, assessed through peer and instructor feedback.	Course project—required analysis of ethical considerations of the required data inputs for the algorithm: in particular, privacy concerns, socioeconomic disparities in access, etc.

	concerns in the age of data-driven power systems engineering.	energy consumption in other communities.		
Demonstrate how to manage large data sets of measurements collected from real-world electric power systems.	Informal assessment of understanding of accessible visualization techniques; stress importance of color-blindness in network models. Class tutorial on handling real-world datasets. Informally assess through Q&A or quick exercises during the tutorial.	Homework assignment: Manipulate and analyze a small real-world dataset, with feedback focused on data handling techniques. Special focus on asking students to identify privacy concerns.	In-class group activity where an inaccessible visualization figure is made more accessible and color-blind friendly. In-class peer review of each other's datasets and in-progress project visualizations, followed by a group discussion on data accuracy and relevance.	Course project—use real dataset from Greensboro, NC to visualize and demonstrate behavior of grid control algorithm.
Design a computational tool that solves optimal power flow problems.	Narrative demonstration of existing tools (e.g., PowerModels.jl or OpenDSSDirect.py) and their features. Informally assess students' understanding through questions.	Homework: Students design basic computational models using the tools demonstrated in class.	Group presentations where students demonstrate their computational tools, assessed by peers and instructor.	Course project—use real dataset described above in software implementation of algorithm in Julia (PowerModels.jl) or Python (OpenDSSDirect.py)
Recognize the the societal impacts of pricing mechanisms in electricity markets.	Demonstration of computing locational marginal prices (LMPs) using the optimal power flow (OPF) model in PowerModels.jl. Discussion of the energy burden metric and how it can be used to assess pricing fairness.	Homework assignment: Differentiate the energy burden function with respect to demand and design a grid upgrade mechanism that systematically reduces energy burden. Analysis of how average income levels, population level, and education level correlate with energy burden.	In-class group activity where real-world census tracts and income data is mapped to a synthetic grid network—most likely the Texas 2k bus test case. A simulation is demonstrated where an extreme weather event produces systemically unfair pricing conditions.	Midterm exam—LMP calculation is assessed. Course project—Students will be encouraged to explore societal applications of the Locational Marginal Burden (LMB) metric as a course project.

List of assignments and weights:

- (0%) Self-assessment puzzles pre-assessment, contains high-level content from each learning objective
- Homework assignments (5-6) total (40%)
  - HW1—Matrix methods for power flow analysis,
  - HW2—Power flow equations, approximations,
  - HW3—Statistical analysis of power flow equations, state estimation theory, dataset handling
  - HW4—Optimization and control with embedded power flow equations
  - HW5—Data-driven and randomized control of power flow equations
  - HW6—Computational techniques
- Mid-semester feedback survey (+1%):
  - Anonymous survey on instruction quality, content, and recommended changes from students
  - Specifically focuses on **the interest and learning goals of the students** to make changes in course content delivery.
  - Close the loop by providing a "response presentation" from instructor to the feedback.
- Midterm exam (to be given between HW4-HW5), (25%)
  - Take home, 48 hours
  - Introduces the merger between randomized analysis of the power flow equations, control, and optimization
  - Project-style, open-ended questions
  - Collaboration permitted on second revision for partial credit
- Final course project (35%)
  - Proposal (5%):
    - 1 page document declaring the interests of the students and team assignments
    - Proposal is due *before midterm exam*, and the midterm exam **will be tailored to the interests of the students** to encourage the development of their projects.
  - Progress report (10%):
    - 3-4 page document declaring the project topic, provides **scaffolding for the project.**
    - Report of methodological developments and/or technical plans.
    - Literature review of related work, identification of relevant resources
    - Potential preliminary results
  - Project report (20%):
    - Option A: Literature review of research state of the art in topic of relevance to the course
    - Option B: Computational application of content covered in course
    - Option C: Preliminary theoretical investigation of new research direction related to topic covered in this course
  - Project presentation (5%):
    - Oral presentation by student pairs at the end of the semester detailing findings, limitations, and future work.

Rationale

The selected topics were chosen to encourage research-oriented thinking in early graduate students. The course content is designed to merge several branches of knowledge into a single, cohesive discipline that is oriented around the mathematical analysis of the grids that power our world. The assessment plan intentionally de-emphasizes exams, in an effort to empower students to develop new knowledge while simultaneously learning the state of the art. The midterm exam, in particular, is a take-home exam designed to create new connections between the studied material and the interests of the students, and will be tailored to the project proposals provided by the students prior to the exams. Formative assessments are incorporated throughout the course to help students track their understanding and receive timely feedback without the pressure of high-stakes grading. The emphasis on project-based learning allows students to apply what they have learned to real-world problems and encourages collaboration and creativity. By allowing students to focus on their individual interests in the final project, the course fosters independent thinking and the ability to connect course material with ongoing research. The goal is to prepare students for both the technical and conceptual challenges they'll encounter in their academic and professional careers.

Computational Power Flow, Sample Exam

ECE 63XX
10/XX/2026

Name: ________________________________

I, ________________________________, commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech Community.

(Revised) **Please read this information:**

- This is a 48-hour take home exam.
- Please do not collaborate during your *first revision* of this exam. You are on your honor.
- Collaboration *may* be permitted for a second revision submission for partial credit.
- Use of generative AI is discouraged due to the nature of the material being creative; reflect on the syllabus policy on use of AI.
- You are responsible for the content of all your answers.
- Please show all your work.
- Please box or circle your final answers.
- This test has 5 problems that total up to 100 points.

Exam wrapper (3 bonus points) (Revised)

Question I. (1 pts)

Reflect on your work in preparation for this course by answering the following questions:

1. Approximately how many hours did you spend studying for this exam? _______
2. Please indicate what percentage of your time was spent on the components of the course:
   (a) Prepared course notes: _______
   (b) Lecture slides and handwritten notes: _______
   (c) Solving and resolving homework: _______
   (d) Researching material on my own: _______

Question II. (1 pts)

Reflect on the topics you believed were your strengths and weaknesses going into this exam. You don't need to use every blank space.

1. Which topic(s) did you feel the most confident about?
   (a) ______________________________________
   (b) ______________________________________
   (c) ______________________________________
2. What topic(s) did you feel the least confident about?
   (a) ______________________________________
   (b) ______________________________________
   (c) ______________________________________

Question III. (1 pts)

Reflect on your interests in this course in preparation for your final project. In your opinion, what was the most interesting part of this course thus far?

____

Item 1: Matrix Methods for Power Flow Analysis (20pts)

Consider the following 3-node electric power network:

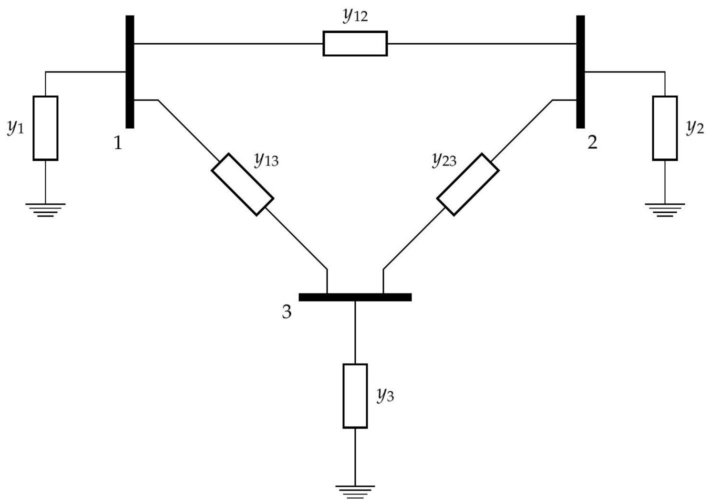

Figure 1: A 3-node electric power network

Question 1a. (10 pts)

Build and write the 3×3 *nodal admittance matrix* Y for this network in terms of the admittance symbols shown for the lines and shunts in Fig. 1.

Solution 1a.

Recall that the nodal admittance matrix is the *graph Laplacian* matrix with the line admittances used to define the graph edge weights. Thus, the matrix Y takes the form

$$Y = \begin{bmatrix} y_1 + \sum_{k \neq 1} y_{1k} & -y_{12} & -y_{13} \\ -y_{21} & y_2 + \sum_{k \neq 2} y_{2k} & -y_{23} \\ -y_{31} & -y_{32} & y_3 + \sum_{k \neq 3} y_{3k} \end{bmatrix} \tag{1}$$

Question 1b. (10 pts)

Derive the power flow equation $s_1 : \mathbb{C}^n \to \mathbb{C}$ for node 1 in the network shown in Fig. 1, which maps the voltages at all nodes in the network to the power injected at node 1. The equation should only depend on the y_{ik}'s and the v_i's.

Solution 1b.

Let $v_i = x_i + \mathrm{j}y_i \in \mathbb{C}$ denote the voltage phasors at each node $i \in \{1,2,3\}$ and let $v = [v_i]_i \in \mathbb{C}^3$ be the network state. Let $y_i \in \mathbb{C}^3$ be the i-th row of the admittance matrix Y. Recall that the power flow equations are

$$s(v) = \mathsf{diag}(v)\underline{Yv},$$

where $\underline{(\cdot)}$ denotes the complex conjugate. Hence, we can write this in elementwise matrix-vector form for the 3-node network shown in Fig. 1 as

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} v_1 & & \\ \cdot & v_2 & \\ \cdot & \cdot & v_3 \end{bmatrix} \underline{\begin{bmatrix} y_1 + \sum_{k\neq 1} y_{1k} & -y_{12} & -y_{13} \\ -y_{21} & y_2 + \sum_{k\neq 2} y_{2k} & -y_{23} \\ -y_{31} & -y_{32} & y_3 + \sum_{k\neq 3} y_{3k} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}}$$

$$= \begin{bmatrix} v_1 & & \\ \cdot & v_2 & \\ \cdot & \cdot & v_3 \end{bmatrix} \underline{\begin{bmatrix} \cdots & y_1^\top & \cdots \\ \cdots & y_2^\top & \cdots \\ \cdots & y_3^\top & \cdots \end{bmatrix} \begin{bmatrix} \vdots \\ v \\ \vdots \end{bmatrix}}$$

Now, we can see that the power flow equations s_i for each node $i \in \{1,2,3\}$ are given as

$$s_i = v_i \underline{y_i^\top v} \quad \text{for all} \quad i \in \{1,2,3\}.$$

Hence, the equation for node 1 is

$$\begin{aligned} s_1 &= v_1 \underline{y_1^\top v} \\ &= v_1 \underline{\left(\left(y_1 + \sum_{k\neq 1} y_{1k}\right) v_1 - y_{12}v_2 - y_{13}v_3\right)} \\ &= v_1 \underline{\left((y_1 + y_{12} + y_{13})\, v_1 - y_{12}v_2 - y_{13}v_3\right)}. \end{aligned}$$

Item 2: Recognize Power Flow Representations (20pts)

Please write in the spaces provided the number that corresponds to the approximation or relaxation of the power flow equations

Question 2a. (5 pts)

Circle your answer: The branch flow model is more appropriate for

1. Single-phase radial distribution networks
2. Single-phase meshed transmission networks

Solution 2a.

The answer is **radial (i.e., tree) distribution networks** because the branch flow model does not enforce consistency in the angle summations around cycles in the network (i.e., the summation of the angles around a cycle must be a multiple of 2π radians for true solutions). **Since transmission network models are typically meshed**, the branch flow model is a relaxation for transmission networks; in contrast, it is *exact* for radial distribution networks.

Question 2b. (5 pts)

Circle your answer: For multi-phase unbalanced radial distribution networks, the branch flow model

1. Is a relaxation of the power flow equations
2. Is an approximation of the power flow equations
3. Is neither a relaxation nor approximation of the power flow equations

Solution 2b.

The answer is that the DistFlow equations are a **relaxation** of the power flow equations for multi-phase *unbalanced* distribution networks, because unbalanced networks have *implicit cycles between phases*; thus, the network cannot be modeled as a single-phase balanced radial network. This is a requirement for the DistFlow equations to be exact; otherwise, it is a relaxation.

Question 2c. (5 pts)

Circle your answer: A node consumes 2 MVA with a leading power factor of 0.8. How much reactive power is the load consuming?

1. 1.2 MVAr
2. -1.2 MVAr
3. 1.6 kVAr
4. -1.6 MVAr

Solution 2c.

The answer is **2, -1.2 MVAR**. The injection is $s = p + jq$ where $|s| = 2MVA$. We can write

$$q = \mathsf{sgn}(q) \cdot \frac{p}{\alpha}\sqrt{1-\alpha^2}$$

where $\alpha \in (0,1)$ is the power factor. Since the power factor is *leading*, $\mathsf{sgn}(q) = -1$ and $p = (0.8) \cdot \left(2 \times 10^6\right) = 1.6 \times 10^6$, so

$$q = -\left(1.6 \times 10^6\right)\frac{\sqrt{1-(0.8)^2}}{0.8} = -1.2 \times 10^6 \text{ VAr} = -1.2 \text{ MVAr}.$$

Question 2d. (5 pts)

For each of the following equations, write TRUE if the equation is either a relaxation or approximation of the power flow equations, and FALSE if it is neither. The equations assume an arbitrary n-node network.

1. $\boldsymbol{p} + \mathrm{j}\boldsymbol{q} = \mathrm{diag}(\boldsymbol{v})\underline{\boldsymbol{Y}\boldsymbol{v}}$, where $\boldsymbol{Y} \in \mathbb{C}^{n\times n}$, $\boldsymbol{p}, \boldsymbol{q}, \in \mathbb{R}^n$ and $\boldsymbol{v} \in \mathbb{C}^n$.
2. $\boldsymbol{v} = \mathbf{1} + \boldsymbol{R}\boldsymbol{p} + \boldsymbol{X}\boldsymbol{q}$, where $\boldsymbol{R}, \boldsymbol{X} \in \mathbb{S}^n_+$, and $\boldsymbol{p}, \boldsymbol{q} \in \mathbb{R}^n$
3. $\boldsymbol{p} = \boldsymbol{B}\boldsymbol{\theta}$, where $\boldsymbol{B} \in \mathbb{S}^n_+$, and $\boldsymbol{p}, \boldsymbol{\theta} \in \mathbb{R}^n$
4. $f_i = \sum_{k=1}^n Y_{ik} v_k$
5. $p_i + \mathrm{j}q_i = v_i \sum_{k=1}^n \underline{Y_{ik}} \underline{v_k} \quad \forall i \in \mathcal{N}$
6. $p_i = |v_i| \sum_{k=1}^n |v_k| \left(G_{ik}\cos(\theta_i - \theta_k) + B_{ik}\sin(\theta_i - \theta_k)\right)$
7. $p_k = \mathsf{trc}(\boldsymbol{H}_k\boldsymbol{W})$, where $\boldsymbol{H}_k := \frac{1}{2}\left(\boldsymbol{Y}^*\boldsymbol{e}_k\boldsymbol{e}_k^\top + \boldsymbol{e}_k\boldsymbol{e}_k^\top\boldsymbol{Y}\right)$ and $\boldsymbol{W} \succeq 0$.
8. $p_k = \mathsf{trc}(\boldsymbol{H}_k\boldsymbol{W})$, where $\boldsymbol{H}_k := \frac{1}{2}\left(\boldsymbol{Y}^*\boldsymbol{e}_k\boldsymbol{e}_k^\top + \boldsymbol{e}_k\boldsymbol{e}_k^\top\boldsymbol{Y}\right)$ and $\mathsf{rank}(\boldsymbol{W}) = 1$.
9. $$\begin{bmatrix} \boldsymbol{p} \\ \boldsymbol{q} \end{bmatrix} = \begin{bmatrix} \boldsymbol{G} & -\boldsymbol{B} \\ -\boldsymbol{B} & -\boldsymbol{G} \end{bmatrix} \begin{bmatrix} |\boldsymbol{v}| - \mathbf{1} \\ \boldsymbol{\theta} \end{bmatrix}$$
10. $p_{ik} = g_{ik}\left(T_{ik} + U_{ik} + |v_i| - |v_k|\right) - b_{ik}\left(\theta_i - \theta_k\right)$, where $U_{ik} \geq \left(|v_i| - |v_k|\right)^2$ and $T_{ik} \geq |\theta_i - \theta_k|^2$ for all $(i,k) \in \mathcal{E}$

Solution 2d.

1. **Relaxations:** 7
2. **Approximations:** 2,3,9,10
3. **Neither:** 1,4,5,6,8

Item 3: Predict Grid Conditions

Question 3a. (20 pts)

A solar photovoltaic device applies a DC voltage v in parallel across two loads with parameters $R_1 := 1\Omega$ and $R_2 := 2\Omega$. Assume that you obtain random measurements from the network of the form

$$f_1 \sim \mathcal{N}(25, 1), \quad f_2 \sim \mathcal{N}(11, 1/4),$$

where f_1 is the current through R_1 and f_2 is the current through R_2. Derive the minimum mean squared error (MMSE) unbiased estimator for v.

Solution 3a.

Let $\boldsymbol{x} = [v]$ be the DC voltage as the state. Applying the power flow equations, the measurement vector is

$$\boldsymbol{h}(\boldsymbol{x}) = \begin{bmatrix} h_1(\boldsymbol{x}) \\ h_2(\boldsymbol{x}) \end{bmatrix} = \begin{bmatrix} \frac{v}{1\Omega} \\ \frac{v}{2\Omega} \end{bmatrix}.$$

The linearization of the measurement operator is

$$\boldsymbol{H} := \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{x}} = \begin{bmatrix} \frac{\partial f_1}{\partial v} \\ \frac{\partial f_2}{\partial v} \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix}.$$

The Gram matrix is

$$\boldsymbol{G} = \boldsymbol{H}^\top \boldsymbol{R}^{-1} \boldsymbol{H} = \begin{bmatrix} 1 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & \cdot \\ \cdot & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 1/2 \end{bmatrix} = 2$$

The MMSE estimator is

$$\hat{\boldsymbol{x}} = \boldsymbol{G}^{-1} \boldsymbol{H}^\top \boldsymbol{R}^{-1} (\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x}_0)) = \frac{1}{2} \begin{bmatrix} 1 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & \cdot \\ \cdot & 4 \end{bmatrix} \begin{bmatrix} 25 \\ 11 \end{bmatrix} = \frac{25 + 22}{2} = 23.5 \text{ V}.$$

Item 4: Economic Power Dispatch

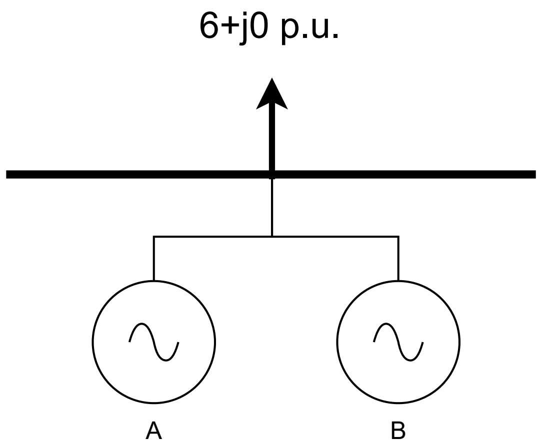

Figure 2: A 1-node network serving load $s = 6 + \mathrm{j}0$ p.u.

Consider a single node network shown in Fig. 2. The cost curves for each generator are given as

$$c_a = \frac{1}{6}p_{g,a}^3 \qquad 0 \leq p_{g,a} \leq 5 \tag{2a}$$

$$c_b = \frac{1}{6}p_{g,b}^3 + 3p_{g,b} \qquad 0 \leq p_{g,b} \leq 5. \tag{2b}$$

Question 4a. (10 pts)

Determine the optimal generations $p_a^{g\star}$ and $p_b^{g\star}$ that minimize the total operating cost.

Solution 4a.

We have that the total operating cost (objective function) of the program $C : \mathbb{R}^2 \to \mathbb{R}$ is given as

$$C(\boldsymbol{p}) = \frac{1}{6}p_{g,a}^3 + \frac{1}{6}p_{g,b}^3 + 3p_{g,b}.$$

The incremental cost vector is

$$\boldsymbol{c} = \nabla_{\boldsymbol{p}} C = \begin{bmatrix} \frac{1}{3}p_{g,a}^2 \\ \frac{1}{3}p_{g,b}^2 + 3 \end{bmatrix} = \begin{bmatrix} \lambda \\ \lambda \end{bmatrix}$$

Moreover, the power balance constraint is given as

$$p_{g,a} + p_{g,b} = 6 \implies p_{g,b} = 6 - p_{g,a}.$$

So, combining the above with the incremental cost vector $\boldsymbol{c}$ yields

$$\frac{1}{3}p_{g,a}^2 = \frac{1}{3}p_{g,b}^2 + 3 = \frac{1}{3}(6 - p_{g,a})^2 + 3.$$

Rearranging, we have

$$\frac{1}{3}p_{g,a}^2 = \frac{1}{3}(6 - p_{g,a})^2 + 3 \implies p_{g,a}^2 = (6 - p_{g,a})^2 + 9 \implies p_{g,a}^2 = 39 - 12p_{g,a} + p_{g,a}^2;$$

thus, we have

$$-39 = -12p_{g,a} \implies p_{g,a} = \frac{39}{12} \implies p_{g,b} = 6 - \frac{39}{12} = \frac{11}{4}.$$

Question 4b. (10 pts)

Determine the locational marginal price (LMP) of power served optimally at the node.

Solution 4b.

The LMP is given as

$$\lambda = \frac{1}{3}p_{g,a}^2 = \frac{1}{3}\left(\frac{39}{12}\right)^2 = 3.52 = \$3,520/\mathsf{MWh}$$

Question 4c. (10 pts)

Determine the total operating cost in $\$/hr$.

Solution 4c.

The total operating cost is

$$C(\boldsymbol{p}^\star) = \frac{1}{6}p_{g,a}^{\star 3} + \frac{1}{6}p_{g,b}^{\star 3} + 3p_{g,b}^{\star} = 17.4375 = \$17,437.50/\mathsf{MWhr}$$

Item 5: Control grid operating conditions

Question 5a. (30 pts)

Consider a single-phase radial distribution network with n nodes. Assume that:

1. There are solar panels installed at every node with a maximum power output of Δ.
2. The LinDistFlow approximation accurately models the grid.
3. Reactive power injections are 0 throughout the grid ($\boldsymbol{q} = \mathbf{0}$).

Derive an upper bound for maximum expected random voltage magnitude perturbation from $\mathbf{1}$ anywhere in the grid, that depends only on the rows of the resistance matrix $\{\boldsymbol{r}_i\}_i$, the number of nodes n, and the maximum solar panel output Δ.

Solution 5a.

Under the problem assumptions, the random voltage magnitude perturbations around $\mathbf{1}$ are given as

$$\boldsymbol{v} - \mathbf{1} = \boldsymbol{R}\boldsymbol{p},$$

where $\boldsymbol{p}$ is a random vector such that $\|\boldsymbol{p}\|_\infty \leq \Delta$ almost surely. Consequently, it follows that $\boldsymbol{p}$ is a sub-Gaussian random vector with parameter at most $\Delta/2$; hence, the maximum random voltage perturbations can be upper bounded as

$$\begin{aligned} \mathsf{E}\left[\max_{i=1,\ldots,n} |v_i - 1|\right] &\overset{(1)}{=} \mathsf{E}\left[\|\boldsymbol{v} - \mathbf{1}\|_\infty\right] \\ &\overset{(2)}{=} \mathsf{E}\left[\|\boldsymbol{R}\boldsymbol{p}\|_\infty\right] \\ &\overset{(3)}{\leq} \|\boldsymbol{R}\|_\infty \, \mathsf{E}\left[\|\boldsymbol{p}\|_\infty\right] \\ &\overset{(4)}{\lesssim} \left(\max_{i=1,\ldots,n} \|\boldsymbol{r}_i\|_2\right) \cdot \left(\Delta\sqrt{\log 2n}\right) \end{aligned}$$

where step (1) is by definition of the ℓ_∞ norm, step (2) is by definition of the LinDistFlow approximation of the power flow equations, step (3) is by the submultiplicative property of any matrix p-norm, and step (4) is by defintion of the matrix ∞ norm and the concentration of the maximum of sub-Gaussian random variables.

ALTERNATIVES:

1. Operator norm-based upper bound is also acceptable.

Instructor: Samuel Talkington **Date Of Lesson:** 09/01/202X

Content Area and Summary: This content is part of the power engineering sub-field of electrical engineering. This lesson aims to empower students to apply techniques from linear algebra and vector calculus to develop *practical* approximations of the power flow equations that can be useful when modeling the grid on a computer.

Course Title and Grade Level: ECE 63XX, Computational and Statistical Power Flow Analysis; intro graduate

Lesson Learning Goals:

The learning goals of this lesson are:

1. Motivate the use of representing complex power as a function of voltage via the power flow equations, in contrast with the linear current-voltage formulations students may be familiar with from undergraduate courses.
2. Introduce students to the matrix-based formulation of the bus injection model of the power flow equations.
3. Introduce students to the concept of the power flow solution manifold.
4. Provide an authentic application of the above two concept items. We will discuss how to apply the above two content items with vector calculus techniques to linearize the power flow equations, producing two practical linear models that can allow for efficient power flow computations.

Assessments:

This lesson is intended to be given early in the term; therefore, it is primarily only relevant to **formative** assessments.

- (1) **In-class puzzle:** In the later half of the lesson, students pair off into groups and discuss how to rewrite the complex-valued power flow equations into polar form.
- (2) **Reproducible programming notebook:** Students are asked to implement the results of the discussion in class in a take-home style assessment, simulating the power flow equations for a general network. Learning technology will be leveraged in this assessment, as students will incorporate this puzzle as a part of a **reproducible programming notebook portfolio** that leverages the Binder reproducible code-sharing platform, which is based off of the Pluto notebook framework for the Julia programming language.
- (3) **Discussion-based assessment:** An additional in-class assessment is enabled by observation of the students response to discussion questions, which will contribute to a participation grade.
- (4) **Project-based assessment:** It is expected that this lesson will be integral to the majority of the course projects developed by students of this course. If this content is relevant to their course projects, students also have an opportunity to demonstrate meeting the above Learning Objectives through said course project(s).

Instructional Strategies and Proactive Management:

- (1) **Early access to content:** The material in this lesson will be made available early to students in an accessible, complete set of lecture notes on Canvas or a similar platform. Expectations for pre-reading will be communicated to students from the beginning of the semester.
- (2) **Use of learning technology:** A key part of this lesson, and the course in general, will be testing the results on real power network data, such as the Texasm A&M University ARPA-e PERFORM synthetic grids, which emulate real power grids in North America. In particular, the Los Alamos National Laboratory PowerModels simulation framework in the Julia programming language will be used. Students will make their implementations of the material available in a reproducible notebook platform such as Binder.
- (3) **Active Student Engagement:** There are multiple opportunities for active students engagement throughout this lesson, in the form of discussion pauses and a pair-and-share exercise.
- (4) **Universal Design for Learning (UDL):** The lesson will incorporate UDL principles. In particular, the lesson will utilize the "Plus-One" framework for inclusive teaching in the following way:
  - A frequent "pinch point" in this lesson is the understanding that the power flow equations are non-linear, in contrast with the linear circuit laws that often have greater emphasis in undergraduate courses which students may have been exposed to.
  - To address this challenge, the instructor will release multiple resources to visualize this non-linearity outside of the traditional mathematical explanation. There will be a 3D-visualization with a simple power grid that shows the non-linearity of the power flow manifold, allowing students to interact with the parameters of a real power grid and get a hands-on look at this property.
  - There will be multiple opportunities to demonstrate understanding of the nonlinearity, which may be application-focused through code, or visual-based through course project presentations.
  - Multiple ways for students to communicate with the instructor and with one another about this concept will be provided around the time this content is introduced.
  - Multiple ways of engaging with the challenges of the non-linearity of the power flow equations will be presented to students, including through society-oriented perspectives (e.g., what can happen to electricity prices if we assume that the equations are linear? Is this fair to all communities involved?)

Interactive Questions:

Convergent Questions:

- (1) What is special about Ohm's law? (Bloom–**Remember**: Linear in the voltages)
- (2) What is special about the complex power equation? (Bloom–**Understand**: Non-linear in the voltages)
- (3) How could we generalize to any n-node circuit, like a power grid? (Bloom–**Apply**: Admittance matrix)
- (4) How can we make the non-linear power flow equations more like Ohm's law? (Bloom–**Analyze**: Taylor series linearization)
- (5) What is special about the practical linear power flow model? (Bloom–**Evaluate**: Defined by Graph Laplacian matrices)
- (6) How can this make a power flow program more practical? (Bloom–**Create**: Replace nonconvex constraints with linear.)

Divergent Questions:

- (1) How does the idea of the nodal admittance matrix relate to other areas of math? (Graph Laplacian matrix)
- (2) What if we want to use rectangular voltage coordinates instead of polar?
- (3) How does the linear power flow model relate with the DC OPF model that is popular in industry?
- (4) What are the potential shortcomings of the linear power flow model?
- (5) Is it possible that the use of linear power flow models in industrial settings could have adverse societal effects?

Beginning Of The Lesson:

Activate prior knowledge:

Review: Linear circuits, Ohm's law, complex power, intro circuit material.
Preview: Generalize linear circuits to n-node circuits.
Hook: Can we use this generalization to model the **entire power grid** with math?

Minute 0-3

Pacing: Slow, asking students to remember circuits concepts from their undergraduate engineering education. Ensuring to cover the fundamentals enough for those who may not have had a power engineering concentration.

Content

Recall from your undergraduate physics class that a resistive circuit is governed by Ohm's law, $V = IR$, where V is voltage, I is current, and R is resistance. The big breakthrough you made in Circuits 2 was that any linear circuit, that is, a circuit with only resistors, inductors, and capacitors, can be represented through a complex-valued form of Ohm's law! We can write

$$V = IZ,$$

where $V, I \in \mathbb{C}$ are complex-valued voltage and current phasors, respectively. Similarly, $Z = R + \mathrm{j}X$ is the complex impedance, where R is the resistance and X is the reactance.

Directions/Activity

Group discussion about how this could be generalized to a larger circuit and what happens if we invert impedance.

The Lesson:

Minute 3-6

Transition: Discuss 1-dimensional circuits before transitioning to n-dimensional graph circuits.

Content

You may remember that there is also a quantity known as the admittance, which is the reciprocal of impedance, i.e.,

$$Y = \frac{1}{Z} = \frac{Z^*}{|Z|^2} = \frac{R - \mathrm{j}X}{R^2 + X^2} = \underbrace{\frac{R}{R^2 + X^2}}_{:=G} + \mathrm{j}\underbrace{\frac{-X}{R^2 + X^2}}_{:=B} := G + \mathrm{j}B,$$

where we call G the conductance and B the susceptance!

Minute 6-10

Pacing:

Transition:

Content(Problem) The complex power flow equations for linear circuits are non-linear in the voltage phasors:

$$S = VI^* = V\left(\frac{V}{Z}\right)^* = \frac{|V|^2}{Z^*}.$$

Directions/Activity

Emphasize how to divide complex numbers, frequent pain point.

Minute 10-19

Pacing: Quick, should be recollection stage still.

Transition: Reminder that circuits are time varying, motivating use of complex numbers.

Content(Problem) The bus admittance matrix generalizes the concept of admittance Y concept to n dimensions, for a circuit with any size. It is a complex valued matrix $\mathbf{Y} \in \mathbb{C}^{n \times n}$ with entries

$$\mathbf{Y}_{ij} = \begin{cases} \sum_{k \neq i} y_{ik} & i = j \\ -y_{ij} & i \neq j \end{cases} \quad \text{for all} \quad i, j = 1, \ldots, n.$$

Directions/Activity

Minute 20-29

Pacing: Quicker, getting up to speed.

Transition: Reminder that circuits are time varying and may have many nodes.

Content(Problem) Similarly to the one-dimensional equations, we can write Ohm's law in vectorized form. First, define complex-valued vectors $\boldsymbol{s}, \boldsymbol{u}, \boldsymbol{f} \in \mathbb{C}^n$, of power injections, voltage phasors, and current phasors, respectively, which have entries of the form

$$\boldsymbol{s} = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix}, \qquad \boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}, \qquad \boldsymbol{f} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix}.$$

Then, Ohm's law can be written for any n-node linear circuit as

$$\boldsymbol{f} = \mathbf{Y}\boldsymbol{u},$$

where $\mathbf{Y} \in \mathbb{C}^{n \times n}$ is the *nodal admittance matrix* that we introduced just a moment ago.

Directions/Activity

Minute 30-34

Pacing:

Transition:

Content: Similarly to the n-node Ohm's law, we can write the n-node complex power injections as a non-linear system of n equations. First, we need to define the operator $\mathsf{diag}(\cdot)$, which forms a *diagonal* matrix out of any vector you supply to the argument:

$$\mathsf{diag}(\boldsymbol{x}) = \begin{bmatrix} x_1 & 0 & \dots & 0 \\ 0 & x_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & x_n \end{bmatrix}, \quad \text{for any} \quad \boldsymbol{x} \in \mathbb{C}^n \quad \text{or} \quad \boldsymbol{x} \in \mathbb{R}^n.$$

Now, we can analyze the entire network's complex power injections via a nonlinear system of equations $\boldsymbol{s} : \mathbb{C}^n \to \mathbb{C}^n$, which takes the form:

$$\boldsymbol{s} = \mathsf{diag}(\boldsymbol{u})\overline{\mathbf{Y}\boldsymbol{u}},$$

where $\overline{(\cdot)}$ denotes the elementwise complex conjugate. This matrix-valued expression captures the complex power injections (pause and remark that this is the bus injection model) for <u>any</u> n-node power network.

Engagement/Active Learning:
Most computer simulation software can't easily represent complex vectors $\boldsymbol{s}$. So, it is often useful to represent the vectorized power flow equations as a function of the <u>polar form</u> of the voltage phasors; that is, a $2n$-dimensional vector containing voltage magnitudes and phase angles: $\boldsymbol{x} = \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{\theta} \end{bmatrix}$.

<u>Pair-and-share (10m):</u> How can you write the vectorized equations in polar form? After a few minutes, the instructor(s) should check for understanding, particularly from students not in this major. The instructor should encourage use of Euler's formula, which is the main technique required for this coordinate transformation.

After hearing from students, the instructor should elaborate that <u>one way</u> to transform the matrix-based power flow equations to polar form is

$$\boldsymbol{s}(\boldsymbol{v}, \boldsymbol{\theta}) = \mathsf{diag}\left(\boldsymbol{v} \circ \exp(\mathrm{j}\boldsymbol{\theta})\right) \overline{\mathbf{Y}\left(\boldsymbol{v} \circ \exp(\mathrm{j}\boldsymbol{\theta})\right)}.$$

The instructor should emphasize that this is <u>not the only way</u> to perform this coordinate transformation. Other approaches used by students, such as representing the equations in trigonometric forms, should be emphasized and discussed, which may potentially lead to the rest of the content being covered in another discussion session.

Minute 35-39

Pacing:

Transition: The instructor explains that students can use the polar coordinate transformation that they developed during the pair-and-share activity to separate the real and reactive power components of the equations as

$$\begin{bmatrix} \boldsymbol{p}(\boldsymbol{v}, \boldsymbol{\theta}) \\ \boldsymbol{q}(\boldsymbol{v}, \boldsymbol{\theta}) \end{bmatrix} = \begin{bmatrix} \mathsf{Re}\left(\mathsf{diag}\left(\boldsymbol{v} \circ \exp(\mathrm{j}\boldsymbol{\theta})\right) \overline{\mathbf{Y}\left(\boldsymbol{v} \circ \exp(\mathrm{j}\boldsymbol{\theta})\right)}\right) \\ \mathsf{Im}\left(\mathsf{diag}\left(\boldsymbol{v} \circ \exp(\mathrm{j}\boldsymbol{\theta})\right) \overline{\mathbf{Y}\left(\boldsymbol{v} \circ \exp(\mathrm{j}\boldsymbol{\theta})\right)}\right) \end{bmatrix}.$$

Content The instructor then asks students to consider what the set of all solutions to the power flow equations might look like. The instructor should emphasize that this is a very mysterious concept that cannot be easily visualized, but they can use the tools they are developing to construct an abstract power flow solution manifold.

Intellectual diversity: The instructor should emphasize that a "manifold" is a mathematical concept that refers to the vector subspace of $\mathbb{C}^n$ that satisfies a nonlinear system of equations. In our case, this is the set of all solutions to the power flow equations $\boldsymbol{s} : \mathbb{C}^n \to \mathbb{C}^n$ for any fixed, or nominal value of voltage phasors $\boldsymbol{u}^\bullet$, which produces the desired power injections $\boldsymbol{s}(\boldsymbol{u}^\bullet)$. The instructor should encourage students to write the equation

$$\mathcal{M}_{\boldsymbol{u}^\bullet} = \{\boldsymbol{u} \in \mathbb{C}^n \ : \ \boldsymbol{s}(\boldsymbol{u}^\bullet) - \boldsymbol{s}(\boldsymbol{u}) = \boldsymbol{0}\}\,.$$

This is practically useful because we can linearize the power flow solution manifold around $\boldsymbol{u}^\bullet$. In particular, we often want to maintain a nominal voltage of $\boldsymbol{u}^\bullet = \boldsymbol{1} + \mathrm{j}\boldsymbol{0}$; this is known as the flat start condition. If we perform a first-order Taylor series linearization around this "flat start" voltage, we obtain

$$\begin{bmatrix} \boldsymbol{p}(\boldsymbol{v},\boldsymbol{\theta}) \\ \boldsymbol{q}(\boldsymbol{v},\boldsymbol{\theta}) \end{bmatrix} \approx \begin{bmatrix} \boldsymbol{p}(\boldsymbol{v}^\bullet,\boldsymbol{\theta}^\bullet) \\ \boldsymbol{q}(\boldsymbol{v}^\bullet,\boldsymbol{\theta}^\bullet) \end{bmatrix} + \begin{bmatrix} \frac{\partial \boldsymbol{p}}{\partial \boldsymbol{v}}(\boldsymbol{v}^\bullet,\boldsymbol{\theta}^\bullet) & \frac{\partial \boldsymbol{p}}{\partial \boldsymbol{\theta}}(\boldsymbol{v}^\bullet,\boldsymbol{\theta}^\bullet) \\ \frac{\partial \boldsymbol{q}}{\partial \boldsymbol{v}}(\boldsymbol{v}^\bullet,\boldsymbol{\theta}^\bullet) & \frac{\partial \boldsymbol{q}}{\partial \boldsymbol{\theta}}(\boldsymbol{v}^\bullet,\boldsymbol{\theta}^\bullet) \end{bmatrix} \begin{bmatrix} \boldsymbol{v}-\boldsymbol{1} \\ \boldsymbol{\theta} \end{bmatrix},$$

simplifying each of the blocks of the Jacobian matrix yields

$$\begin{bmatrix} \boldsymbol{p}(\boldsymbol{v},\boldsymbol{\theta}) \\ \boldsymbol{q}(\boldsymbol{v},\boldsymbol{\theta}) \end{bmatrix} \approx \begin{bmatrix} \boldsymbol{G} & -\boldsymbol{B} \\ -\boldsymbol{B} & -\boldsymbol{G} \end{bmatrix} \begin{bmatrix} \boldsymbol{v}-\boldsymbol{1} \\ \boldsymbol{\theta} \end{bmatrix},$$

where $\boldsymbol{G}$, $\boldsymbol{B}$ are the real and imaginary components of the nodal admittance matrix $\boldsymbol{Y}$, respectively!

Directions/Activity Intellectual diversity: The instructor will ask if any students in the class are returning to school from the Power Engineering industry. If such students are present, the instructor will ask if this looks like the familiar "DC" power flow approximation that is used in industrial settings.

Closure:

Content Summary: To summarize, we introduced a powerful way to understand the non-linear power flow equations through practical linear approximations.

The lesson will conclude with a group discussion of the benefits and challenges of such a model, with a particular focus on the **societal risks** associated with using this model–e.g., how can linearizing the power flow equations lead to unfair outcomes in electricity prices? Or, how can it lead to increased risks of power blackouts and costs to communities, in exchange for operational efficiency? Is there a better way we can do things? In contrast, what are the benefits and improvements gained from this approach?

ECE 63XX:
Computational Power System Analysis

XYZ Universty

Fall 2026

Instructor: Samuel Talkington | E-mail: talkington@gatech.edu
Web: talkington.dev/teaching

Class Room: Van Leer XX | **Class Hours:** T/Th 5-6:15pm
Instructor Office: Van Leer C248 | **Office Hours:** Th/F 11:30am-1pm

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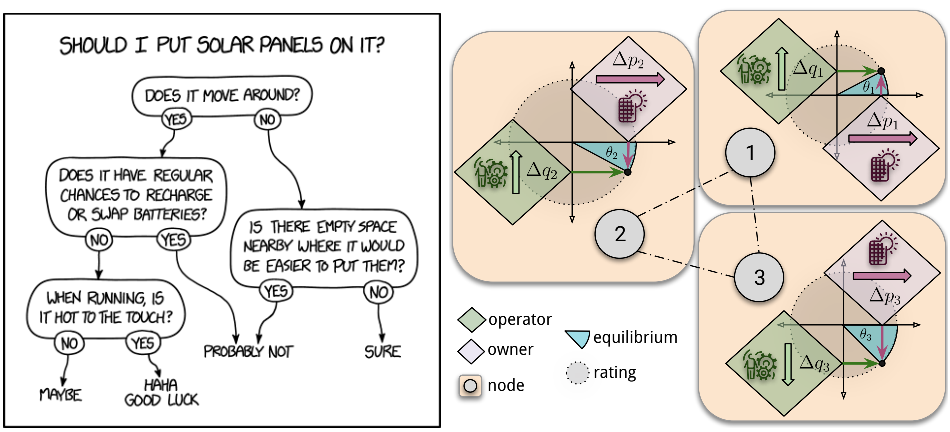

Simplified and detailed illustrations of what we will learn in the class (leftmost courtesy of xkcd).

Course Description

In this course, we will embark on a journey to unravel the *power flow equations*—the fundamental laws that govern our electric power grid. Despite more than a century of study, many properties of these non-linear equations present captivating engineering challenges. To this day, just counting the total number of solutions is still an unsolved mystery. Together, we will dive deep into the mathematical structure of these equations, explore their various representations, and learn how to harness scientific computing techniques to find practical solutions that power our world.

Learning Objectives

By the end of this class, you will be well-prepared to **computationally model power flow** in electrical networks and integrate these models into engineering decision-making processes. We will have a special emphasis on modern appplications of interest, like optimization, control, and statistical analysis. These skills are crucial for addressing present and future real-world challenges such as integrating renewable energy sources, optimizing the efficiency of the power grid, and bolstering the grid's resilience to unexpected extremes.

Moreover, we will explore surprising and exciting connections between power systems and broader mathematical fields like graph theory, high dimensional statistics, and optimization. At the end of this course, you will be well-prepared for advanced studies in smart grid technologies, sustainable energy systems, and interdisciplinary research beyond the classroom.

Upon successful completion of this course, students should be able to:

- Construct matrix-based network models for electric power systems.
- Recognize the appropriateness of using different representations of the power flow equations to perform a power flow analysis.
- Develop optimization and control programs whose solutions are ensured to satisfy the power flow equations.
- Predict future grid operating conditions by constructing and testing statistical grid models.
- Summarize the ethical considerations of different optimization and statistical models for power flow.
- Manage large data sets of measurements collected from electric power systems.
- Design a computational tool that solves optimal power flow problems.

Resources

- **Our course notes:** Will be made periodically available on Canvas.
- **Main textbooks (available for free online):**
  - Steven H. Low, "Power System Analysis: Analytical tools and structural properties", Caltech, 2024. Link.
  - Daniel K. Molzahn, "A Survey of Relaxations and Approximations of the Power Flow Equations", Foundations and Trends in Electric Energy Systems, 2018. Link.
- **Additional resources:** This is a collection of free textbook resources that I have found useful over the years. Our studies together will only use a subset of this material, and these books are *not* required for this course. We will go over anything we need from these books together, but these are great places to start your research career.
  - Roman Vershynin, "High Dimensional Probability: An Introduction with Applications in Data Science", Cambridge University Press, 2019. Link.
  - Stephen Boyd and Lieven Vandenberghe, "Convex Optimization", Cambridge University Press, 2004. Link.

Prerequisites/Corequisites

Familiarity with the core elements of these courses will be assumed and central to this class:

- Undergraduate circuit analysis, sometimes known as 'Circuits 1 and 2".
- Undergraduate matrix theory and linear algebra.

The following skills are helpful for ensuring success in this course, but are *not* prerequisites:

- Probability theory and statistical inference.
- Linear and convex optimization.
- Linear systems and controls.

Course Structure

Problem sets

There will be a problem set assigned approximately every 1-2 weeks. Problem sets are intended to assess both basic knowledge of the course material and to encourage a deeper understanding, so it is likely that some additional research will be required beyond coming to class. Each problem in a problem set will be graded from 0-2 for understanding and completion. Credit will be assigned as follows:

- Solution was complete and correct: 2/2
- Solution was almost completely correct with a minor technical error 1.5/2
- Solution showed solid grasp of the problem, and was partially correct: 1/2
- Solution was partially attempted but was incomplete: 0.5/2
- Solution could not be understood or was not attempted: 0/2

The minimum of your problem set scores will not be considered in your final grade.

Participation puzzles

Beginning the second week of class, *participation puzzles* may occasionally appear in class. During a puzzle, a challenging, counter-intuitive problem will be presented in a fun or "game-ified" manner. The puzzle will be followed by a period of group discussion and peer feedback, and then reviewing the solution together. Puzzles will be graded on a binary scale, where full credit is earned for any submission. Per the name, you will always receive full credit for participating. The goal of the puzzles is to promote outside-the-box thinking, promote a community atmosphere, and encourage in-person attendance. I expect all students to receive full points in this section.

Midterm exam

There will be one take-home midterm exam given halfway through the course. It will be available for 48 hours. *The midterm exam may be revised*; this opportunity is intended to encourage a growth mindset and to serve as a redemption opportunity for those who wish to improve their final grade. If you choose to revise your midterm exam, you must submit a 1 page reflection statement.

Assignments

Credit will be awarded for the work you do according to the following distribution:

- **0%**: Self-assessment
- **2%**: Participation puzzles
- **48%**: Take-home problem sets (x5-6, percentages evenly distributed)
  - HW1: Matrix methods for power flow analysis
  - HW2: Power flow equations and approximations
  - HW3: Statistical analysis of power flow and state estimation
  - HW4: Optimization and control with embedded power flow equations
  - HW5: Data-driven and randomized control of power flow
  - HW6: Computational techniques and dataset handling
- **30%**: Final class project
  - **0%**: Project proposal/interest statement
  - **5%**: Progress report
  - **20%**: Final report
  - **5%**: In-person project presentation
- **20%**: Midterm exam + revisions

Credit will be recorded on Canvas. Please contact me if anything on Canvas is incorrect. Your grade will be calculated based on the following credit thresholds:

$$A \geq 90.0\%, \quad B \geq 80.0\%, \quad C \geq 70.0\%, \quad D \geq 60.0\%, \quad F < 60\%.$$

Required materials

This is a computational and theoretical course, so the only required materials are a pen, a notebook, and a computer. However, we will be performing simulations using the Julia programming language in this course, which is a compiled programming language. Compiled programming languages can sometimes be a bit more resource-intensive than scripting languages like Python. The ECE Laptop Loaner program can provide you with a computer for the semester if your computer is not powerful enough; please reach out to me if I can help you on this front.

Course Expectations and Guidelines

Support for student health and well-being

I would like to ask that you please be kind to yourself. Your physical and mental health matter. I affirm my support for your well-being and I am thrilled that you have joined us to learn something new together this semester.

Communication

There will be a Piazza section set up and linked to Canvas. That is the preferred place to ask technical questions so that everyone in the class can see the answer (or answer themselves) and ask follow-up questions in the same place. Find our class signup link at: `https://piazza.com/asdf`

Attendance and participation

Lecture attendance is expected unless you have a compelling reason not to do so, but you are fully trusted and your time is respected. If you miss a lecture with a valid excuse, feel free to email me to check what you missed that day. In particular, excused absences include, but are *not limited to* religious or cultural observations, physical or mental health matters, job search obligations, or housing instability.

Late Work Guidelines

- **Homework:** Extensions will be made on a case-by-case basis. Without prior arrangements, it cannot be submitted late. After solutions are released, late submissions will not be accepted. Please email me with excused delays so that we can work out submission details.
- **Midterm:** The midterm exam is take home, but if you are sick or unable to take the exam during the time period, you will be able to take a make-up midterm. The make-up exam may be different than the original.
- **Project:** The final project is due by the final day of the term, as such, this is a hard deadline.

Collaboration and group work

Students are *strongly* encouraged to discuss and collaborate on homework problems with one another. However, each student must individually produce and turn in their own solutions written in their own words. Cases where solutions appear to be identical or nearly identical will be immediately referred to the Office of Student Integrity.

Use of Artificial Intelligence

Use of Artificial Intelligence as a collaborator in your study is acceptable, but cautioned. Additionally, please note that your AI collaborator is not you. The above Collaboration and Group Work policy applies to AI in the same way as a human—you must type up all solutions in your own words. Cases of clear plagiarism, such as hallucinated or unrelated content, will not receive credit.

Academic Integrity and Honesty

At Georgia Tech, we believe that it is important to strive for an atmosphere of mutual respect, acknowledgement, and responsibility between faculty members and the student body. See `http://www.catalog.gatech.edu/rules/22/` for an articulation of some basic expectations that you can have of me and that I have of you. In the end, simple respect for knowledge, hard work, and cordial interactions will help build the environment we seek. Therefore, I encourage you to remain committed to the ideals of Georgia Tech while in this class.

Accommodations for Students with Disabilities

If you are a student with learning needs that require special accommodation, contact the Office of Disability Services at `(404) 894-2563` or `http://disabilityservices.gatech.edu/` as soon as possible to make an appointment to discuss your special needs and to obtain an accommodations letter. In addition, if you are a student with **visible or invisible** challenges, please know that you can reach out to me to discuss your learning needs, so that I can best serve you.