Symmetrical Components

ECE 4320

04/02/2025

Agenda:

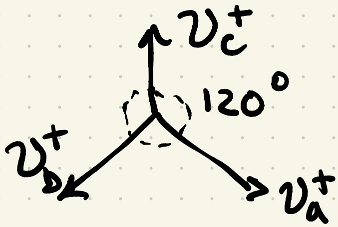
- Analysis/Math
- Sequence-to-phase transformation
- Examples

→ Efficient to analyze multiphase unbalanced networks

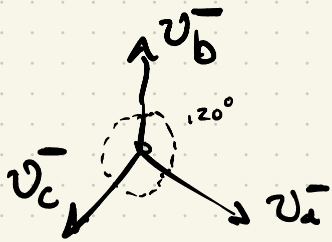
IDEA: Decompose a multiphase unbalanced electrical quantity (or system) into three sequence networks or "components"

Bergen Vittal chapter 12

* **Positive sequence**
  superscript "+" or "1"
  - $|V_a^+| = |V_b^+| = |V_c^+|$
  - $V_b^+ = V_a^+ e^{-j(120°)}$
  - $V_c^+ = V_a^+ e^{+j(120°)}$

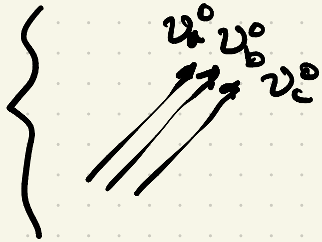


* **Negative sequence** "−" or "2"
  - $|V_a^-| = |V_b^-| = |V_c^-|$
  - $v_b^- = v_a^- e^{j(120°)}$
  - $v_c^- = v_a^- e^{j(-120°)}$



Zero Sequence: superscript "o"

$\hookrightarrow v_a^0 = v_b^0 = v_c^0$



Claim: Any arbitrary set of 3 voltage phasors can be represented as a sum of the 3 sequence components ("+", "−", "o")

$$\left.\begin{aligned} V_a &= V_a^0 + V_a^+ + V_a^- \\ V_b &= V_b^0 + V_b^+ + V_b^- \\ V_c &= V_c^0 + V_c^+ + V_c^- \end{aligned}\right\}$$

How many degrees of freedom?

Count the degrees of freedom

Phase Coordinate

$|V_a|, |V_b|, |V_c|,$
$\angle V_a, \angle V_b, \angle V_c$

Sequence Coordinates

for any particular phase: e.g. a:

$|V_a^+|, |V_a^-|, |V_a^0|$
$\angle V_a^+, \angle V_a^-, \angle V_a^0$

Dictates the values of other Phases

1.) Pick a phase (e.g. a)

2.) Sequence - to - phase transformation (construct it)

Define $\boxed{\alpha = 1\angle 120^\circ = e^{j(120^\circ)}}$

Observe:

$\alpha^3 = (1\angle 120^\circ)(1\angle 120^\circ)(1\angle 120^\circ) = 1$

$\alpha^3 + \alpha^2 + \alpha = 0.$ (Balanced phasors sum to zero!)

$\alpha^2 = \alpha^*$

Problem: Given v_a^+, v_a^-, v_a^0, construct v_a, v_b, v_c:

$$\underbrace{\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}}_{V} = \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} v_a^0 \\ v_a^+ \\ v_a^- \end{bmatrix}}_{V_3}$$

$$V = AV_3$$

Phase-to-Sequence Transformation

Frederik Geth

$$V_S = \underbrace{A^{-1}}V$$

$\det(A) \neq 0$

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$$

* Drop subscript "a" in sequence coordinates

→ Assumed that phase "a" is implied (arbitrary)

Symmetrical Comp. Examples:

* Balanced, pos. seq.

$$V = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 10\angle -120^\circ \\ 10\angle +120^\circ \end{bmatrix}$$

Phase-to sequence

seq. comp.

$$V_S = A^{-1}V = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 10\angle 0^\circ \\ 10\angle -120^\circ \\ 10\angle +120^\circ \end{bmatrix}$$

$$\Rightarrow V_S = \begin{bmatrix} 0 \\ 10\angle 0^\circ \\ 0 \end{bmatrix}$$

Only positive sequence!