Georgia Tech College of Engineering
School of Electrical and Computer Engineering

ECE 4320: State Estimation

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Logistics

Agenda: next 2 weeks

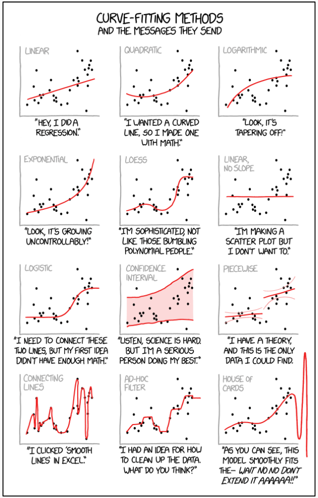
- State estimation (x2)

Office hours w/ me

- Project, coding, support
- Research interest chats

This week: Thurs. 2pm;

Next week: TBD.



Source: xkcd

Recap

Last time:

- We introduced **unit commitment**–an extension of DC Optimal Power Flow (OPF) with **discrete** decision.
- We talked about how to solve Unit Commitment with the “branch-and-bound” tree.

Today:

- We’ll talk about *state estimation*–bringing the world of statistics to power systems.
- Time permitting, we’ll also talk about how to solve these problems computationally.

Overview

Introduction to State Estimation

Mathematical Formulation

Solution Methods

Linear approximation for state estimation and simple example

Introduction to State Estimation

What is State Estimation?

- Process of estimating the state variables (voltage phasors) of a power system using measurements.
- Essential for real-time monitoring, control, and operation of power systems.
- Objective: Obtain the best estimate of the state of the system given the available measurements.

Mathematical Formulation

System Model

- **State vector**: $\boldsymbol{x} \in \mathbb{R}^n$ representing voltage angles and magnitudes:

$$\boldsymbol{x} = \begin{bmatrix} \theta_2 & \theta_3 & \dots & v_1 & v_2 & \dots \end{bmatrix}^\mathsf{T}$$

- **Measurement function**: $\boldsymbol{h}(\boldsymbol{x})$, a *nonlinear* mapping from states to measurement, consisting of power flows, injections, or bus voltages.

Mathematical formulation

Idea: Estimate x from noisy observations of $h(x)$:

- Measurement vector $\boldsymbol{z} \in \mathbb{R}^m$:

$$\boldsymbol{z} = \boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{\epsilon}$$

- $\boldsymbol{\epsilon}$: Measurement noise. We'll model this as a standard Gaussian vector:

$$\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}).$$

The measurement model

$$\boldsymbol{z} = \begin{bmatrix} z_1 \\ \vdots \\ z_m \end{bmatrix} = \begin{bmatrix} h_1(\boldsymbol{x}) \\ \vdots \\ h_m(\boldsymbol{x}) \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \vdots \\ \epsilon_m \end{bmatrix}$$

Solution Methods

Least squares

Least squares estimation

- Define the *residuals* (estimation error) as: $z_i - h_i(\boldsymbol{x})$.
- *Least squares* seeks to minimize the sum of the *squared residuals*:

$$\min_{\boldsymbol{x}} \sum_{i=1}^{m} (z_i - h_i(\boldsymbol{x}))^2 .$$

Motivation for Weighted Least Squares (WLS) Estimation

- What if we **trust some measurements more** than others?
- **Example:** What if the measurement noise variance is different at each bus?

$$\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{R}),$$

where $\boldsymbol{R}$ is the *covariance martrix*:

$$\boldsymbol{R} = \begin{bmatrix} \sigma_1^2 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma_n^2 \end{bmatrix}.$$

- The measurements are still: $\boldsymbol{z} = \boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{\epsilon}$.

Idea: Weighted least squares

Minimize the **weighted** sum of the squared residuals:

$$\min_{\boldsymbol{x}} \sum_{i=1}^{m} w_i \left(z_i - h(\boldsymbol{x})\right)^2$$

Pair and share:

What should we set the weights w_i to be?

WLS optimization problem

$$\underbrace{\min_{\boldsymbol{x}} \sum_{i=1}^{m} \frac{(z_i - h(\boldsymbol{x}))^{\color{red}{2}}}{\sigma_i^2}}_{=C(\boldsymbol{x})} = \min_{\boldsymbol{x}} (\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x}))^{\mathsf{T}} \boldsymbol{R}^{-1} (\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x}))$$

Solving WLS

- **How to solve?**: Iterative methods (e.g., Gauss-Newton, Newton-Raphson).
- Convergence depends on good initial estimates.
- Solve by setting the derivatives of $C(\boldsymbol{x}) = 0$:

$$\nabla_{\boldsymbol{x}} C = \boldsymbol{g}(\boldsymbol{x}) =$$

Derivation of the Newton-Raphson solution

Gain matrix

Linear approximation for state estimation and simple example

Linear State Estimation (DC Model)

- Simplification of the AC model by linearizing the measurement model.
- The measurement function becomes linear:

$$\boldsymbol{z} = \boldsymbol{H}\boldsymbol{x} + \epsilon$$

3-bus system setup

Next time

Next time

- Observability analysis
- Pseudo-measurements
- New extensions of state estimation