Georgia Tech College of Engineering
School of Electrical
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ECE 4320: Unit Commitment

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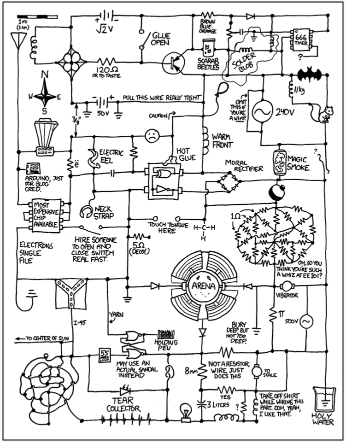
Logistics

Agenda: next 2 weeks

- Unit commitment
- State estimation (x2)

Office hours w/ me

- Project, coding, support
- Research interest chats



Source: xkcd

Recap

Last time:

- We introduced **optimal power flow**–make sure to check Prof. Molzahn's notes!
- We produced an *approximation* of this problem, **DC optimal power flow**.

Today:

- We'll extend DC optimal power flow to handle new practical challenges.
- This extension is called the **DC *unit commitment* problem.**

Handling multiple time periods

"Day-ahead" scheduling

- In DCOPF, we found generations $\boldsymbol{p}_g$ for a **single snapshot in time**.
    - What are some potential challenges with this?
    - Large generators can take time (hours) to start up and shut down.
    - In practice, we need to schedule their generation "day-ahead".

The key ideas behind unit commitment: Questions

Unit commitment combines two key ideas:

- **Key idea #1:** We allow the loads to **be time varying** over time periods $t = 1, \ldots, T$:

$$p_d^1, \ldots, p_d^t, \ldots, p_d^T.$$

  - Q: How do we handle this?
- **Key idea #2:** We need to allow generators to be able to **turn off and on**.
  - Q: How do we handle this?

The key ideas behind unit commitment: Answers

Unit commitment combines two key ideas:

- **Key idea #1:** We allow the loads to **be time varying** over time periods $t = 1, \dots, T$:

$$p_d^1, \dots, p_d^t, \dots, p_d^T.$$

  - Q: How do we handle this?
  - A: We need to find generator setpoints over these time periods:

$$\underline{p_g^1, \dots, p_g^t, \dots, p_g^T}.$$

- **Key idea #2:** We need to allow generators to be able to **turn off and on.**
  - Q: How do we handle this?
  - A: Introduce a new **binary variable** for each generator, at each time period:

$$u_i^t = \begin{cases} 1 & \text{if generator } i \text{ is } \textbf{on} \text{ at time } t \\ 0 & \text{if generator } i \text{ is } \textbf{off} \text{ at time } t. \end{cases}$$

Recall: DC optimal power flow approximation

DC optimal power flow approximation

$$\min_{\boldsymbol{\theta}, \boldsymbol{p}_g} \quad \sum_{i=1}^{n} c_i\left(p_{gi}\right)$$

net costs of all generation

$$\text{s.t.} \quad p_{gi} - p_{di} = \sum_{k=1}^{n} B_{ik}\left(\theta_i - \theta_k\right)$$

Linear approx (DC power flow)

$$p_{gi}^{\min} \leq p_{gi} \leq p_{gi}^{\max}$$

generation limits

$$-P_{ik}^{\max} \leq B_{ik}\left(\theta_i - \theta_k\right) \leq P_{ik}^{\max}$$

flow limits

Extending DCOPF → Unit Commitment

We need to **modify and add constraints** to our DC OPF problem to handle these two key ideas!

Constraints we modify

Let's start transforming the DC OPF constraints, using our **binary variables** $u_i^t \in \{0,1\}$:

Example: transform generator limits

(*)

$$p_{gi}^{\min} \leq p_{gi} \leq p_{gi}^{\max} \quad \overset{?}{\longrightarrow}$$

$u_i^t = 1 \Rightarrow$ generator i "can be" on

$u_i^t = 0 \Rightarrow$ generator i "must be" off

$$u_i^t p_{gi}^{\min} \leq p_{gi} \leq u_i^t p_{gi}^{\max}$$

Constraints we add

We also need to add **totally new** constraints:

1: Ramp rates

2: Minimum up time

3: Minimum down time

4: On/off status

gen. $P_{g_i}^{(t)}$ can't change too much from $t-1$

$$\left|P_{g_i}^{(t)} - P_{g_i}^{(t-1)}\right| \leq R_i + M\left((1-u_i^t) + (1-u_i^{t-1})\right)$$

(M: big-M)

Constraint disabled if gen turned on/off

$$u_i^{t+K} \geq u_i^t - u_i^{t-1}, \quad K = 1, \ldots, \min(T-t, U_i)$$

(U_i: minimum uptime)

$$u_i^{t+K} \leq 1 + u_i^t - u_i^{t-1}, \quad K = 1, \ldots, \min(T-t, D_i)$$

(D_i: min downtime)

$$u_i^t \in \{0, 1\}$$

Putting it all together: DC Unit Commitment (DCUC)

$$\min \sum_{t=1}^{T} \sum_{i=1}^{n} C_i(P_{gi}^{(t)}) + C_i^0 u_i^t$$

(gen. limits) $$u_i^t P_{gi}^{min} \leq P_{gi}^t \leq u_i^t P_{gi}^{max}$$

vector

(Power bal.) $$P_g^t - P_d^t = B\theta^t$$

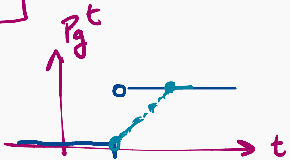
(Line flow limits) $$-P_{iK}^{max} \leq P_{i,K}^t \leq P_{iK}^{max}$$

$\vdots$

(other new constraints)

Other possible additions to DCUC you may see in practice:

- Start-up costs
- Shut-down costs
- Reserve requirements → certain amount of excess generation
- Startup/shutdown/production rates



Wait a minute, something seems fishy...

For each generator $i = 1, \ldots, n$ and each time period $t = 1, \ldots, T$, we have two possible choices:

$$u_i^t \in \{0, 1\}.$$

- **Example:** Suppose we want to schedule $n = 11$ generators over $T = 24$ hours.
- **How many combinations of the u_i^t's are possible?**

2^{nT}

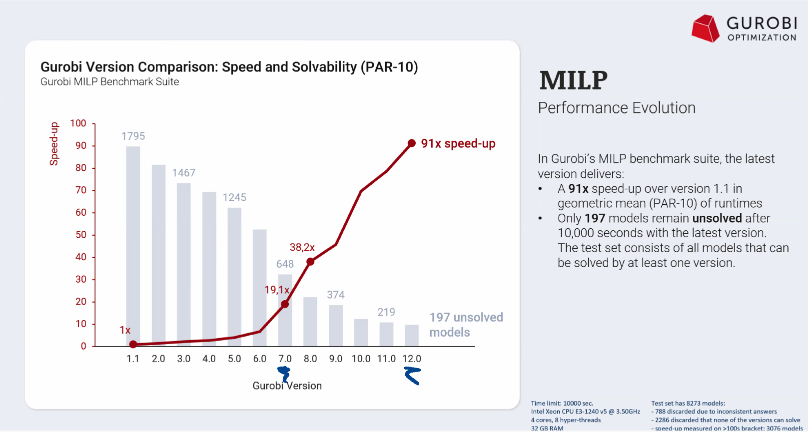
The estimated number of atoms in the known universe

100,000,000,000,000,000,000,000,000,
000,000,000,000,000,000,000,000,000,
000,000,000,000,000,000,000,000,000

Hint: it's about that much ↑

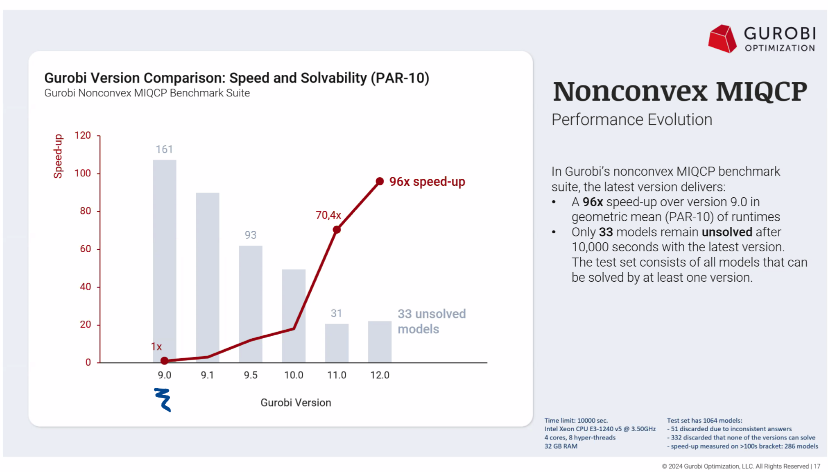
Aside: Mixed-Integer Linear Programming

- DCUC is a *mixed-integer linear program* (**MILP**).
- There has been a **ton of progress** in solving these problems in recent years!



The future: Mixed-Integer Non-linear Programming

- These days, we're making progress on mixed-integer **non-linear** programming, too!
- These speed-ups are **more** than just faster computers.
- Could *you* make the next breakthrough?

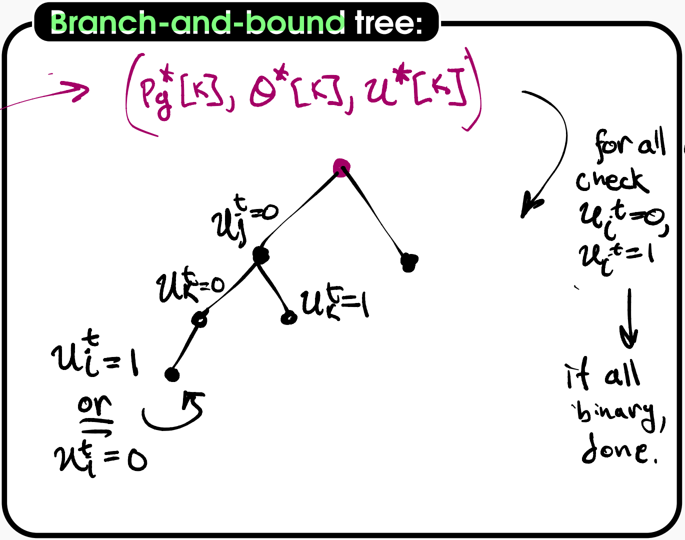


MILP DC Unit Commitment

"Relax" the binaries:

$u_i^t \in \{0,1\} \rightarrow u_i^t \in [0,1]$,

into continuous variables.



When we're done: Branch-and-bound

"Relax" the binaries:

$$u_i^t \in \{0,1\} \rightarrow u_i^t \in [0,1],$$

into continuous variables.

Termination conditions and optimality:

Solution to the relaxed problem @ each node lower bounds the true optimal solution

Meaning, @ node K

$$\sum_{t=1}^{T} \sum_{i=1}^{n} c(P_{g_i}^t) + c_i^0 u_i^t$$

$$= f(P_{g_i}^t, u_i^t) \leq f(P_{g_i}^{t*}, u_i^{t*})$$

Thanks

Thank you so much for attending!

- Please consider giving me feedback on this brief survey!
- I would truly value your input on my teaching so I can better serve you.

Additional resources

Click the links below for useful resources on unit commitment:

- A Brief History of Linear and Mixed-Integer Programming Computation
- Tutorial slides on Unit Commitment
- Bernard Knueven, James Ostrowski, Jean-Paul Watson (2020) On Mixed-Integer Programming Formulations for the Unit Commitment Problem. INFORMS Journal on Computing 32(4):857-876.

Click the link below for one of the *latest breakthroughs* on unit commitment:

- Dominic Yang, Bernard Knueven, Jean-Paul Watson, James Ostrowski, Near-optimal solutions for day-ahead unit commitment, Electric Power Systems Research,Volume 234, 2024, 110678