ECE 4320, Spring 2025
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Homework 5
Due: April 22

4 problems, 100 points possible

1 State Estimation

This problem tasks you with formulating (but not solving) the equations associated with a state estimation problem for the two-bus system shown below in Fig. 1.

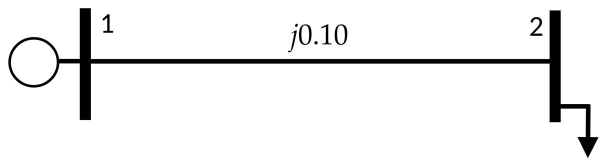

Figure 1: Two bus system

This system has a generator at bus 1 and a load at bus 2. The line is modeled with a series impedance of $j0.10$ per unit. Bus 1 sets the angle reference so $\theta_1 = 0°$. (Note: the phase angle reference is not a measurement, so θ_1 is modeled as an exact value in the formulation below.) For this system, the state vector $\boldsymbol{x}$ contains the voltage angle at bus 2 (θ_2), the voltage magnitude at bus 1 (V_1), and the voltage magnitude at bus 2 (V_2):

$$\boldsymbol{x} = \begin{bmatrix} \theta_2 \\ V_1 \\ V_2 \end{bmatrix}$$

Table 1: One sample of measurements of the system

Measurement	**Measured Value (per unit)**	**Standard Deviation (per unit)**
Voltage magnitude at bus 1, V_1	$V_1 = 1.001$	$\sigma_1 = 0.01$
Voltage magnitude at bus 2, V_2	$V_2 = 0.969$	$\sigma_2 = 0.01$
Active power flow on the line, P_{12}	$P_{12} = 1.033$	$\sigma_3 = 0.03$
Reactive power flow on the line, Q_{12}	$Q_{12} = 0.464$	$\sigma_4 = 0.03$

Problem 1 (25pts)
Write the (weighted) least-squares optimization formulation for the state estimation problem that uses the four specified measurements from the table to compute an estimate for the system state $\boldsymbol{x}$. **Note: You do not need to solve this formulation.**

1. Write down the measurement vector $\boldsymbol{z}$ as described in Table 1 and construct the measurement functions $\boldsymbol{h}(\boldsymbol{x}) = \begin{bmatrix} h_1(\boldsymbol{x}) & \dots & h_4(\boldsymbol{x}) \end{bmatrix}^\top$. [15pts]

2. Write the objective function $C(\boldsymbol{x})$ for the least squares problem. [10pts]

2 Matrix Methods for Three-Phase Fault Analysis

Consider the admittance matrix shown below, with values given in per unit.

$$\boldsymbol{Y} = \begin{bmatrix} 0+j22 & 0-j8 & 0-j4 & 0-j10 \\ 0-j8 & 0+j8 & 0+j0 & 0+j0 \\ 0-j4 & 0+j0 & 0+j9.5 & 0-j5.5 \\ 0-j10 & 0+j0 & 0-j5.5 & 1-j12.5 \end{bmatrix}$$

Problem 2 (25pts)

From the admittance matrix $\boldsymbol{Y}$, complete the following tasks:

1. Draw the one-line diagram for the power system that has the admittance matrix $\boldsymbol{Y}$ shown above. On your one-line diagram, label all **impedances** in per-unit representation. [6pts]
2. Compute the impedance matrix $\boldsymbol{Z} = \boldsymbol{Y}^{-1}$ using MATLAB, or your computation method of choice. [3pts]
3. Suppose that a *bolted three-phase fault* occurs at bus 3. Neglect all prefault current and assume a flat prefault voltage of $1\angle 0°$ per unit. Compute the following using the impedance matrix method:
   (a) The current into the fault in per unit. [8pts]
   (b) The voltages during the fault at buses 1, 2, and 4. [8pts]
4. **[BONUS: +3pts]** Explain how to make it impossible to compute $\boldsymbol{Z}$ (i.e., make $\boldsymbol{Y}$ singular) by changing exactly 1 entry of $\boldsymbol{Y}$. Explain what this means physically in 1 or 2 sentences.

3 Analysis of Unbalanced Faults

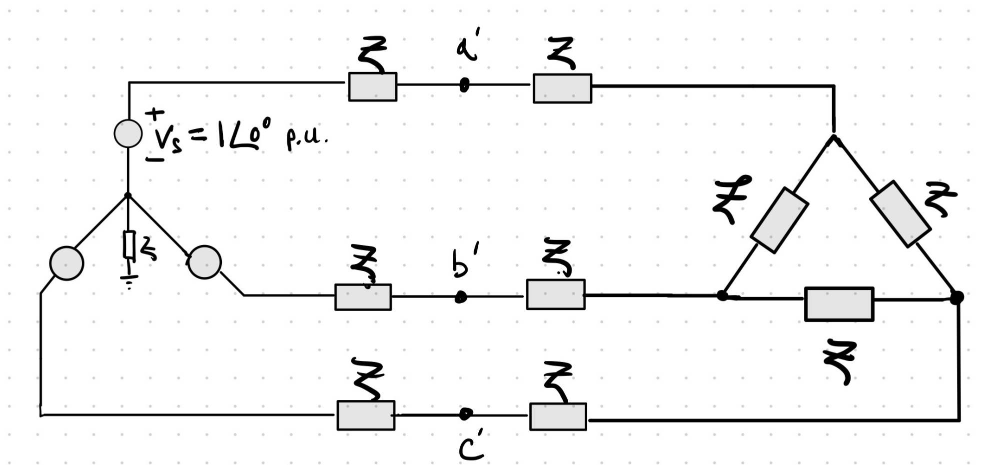

Figure 2: The Problem 3 system

Consider the system shown in Fig. 2. Assume that the **voltage source provides balanced positive sequence voltage phasors**, with the phase a voltage phasor $V_s = 1\angle 0°$ per unit. Also assume that the impedances are equal to $Z = 0 + j1$ per unit. Observe that the wye-connected voltage source is grounded through an impedance Z.

Problem 3 (25pts)

For the system shown in Fig. 2, compute each of the following fault types. Assume that each of the faults is *bolted* (that is, zero fault impedance to ground).

1. Compute the fault current $I_{a'f}$ for a bolted fault from a' to ground (single line to ground fault on phase a). [10pts]
2. Compute the fault current flowing from b' to ground ($I_{b'f}$) for a bolted fault from b' and c' to ground (double line to ground fault on phases b and c). [10pts]
3. Compute the current flowing from phase a to ground during a three-line to ground bolted fault at points a', b', and c'. [5pts]

4 Forming positive- negative-, and zero-sequence circuits from a one-line diagram

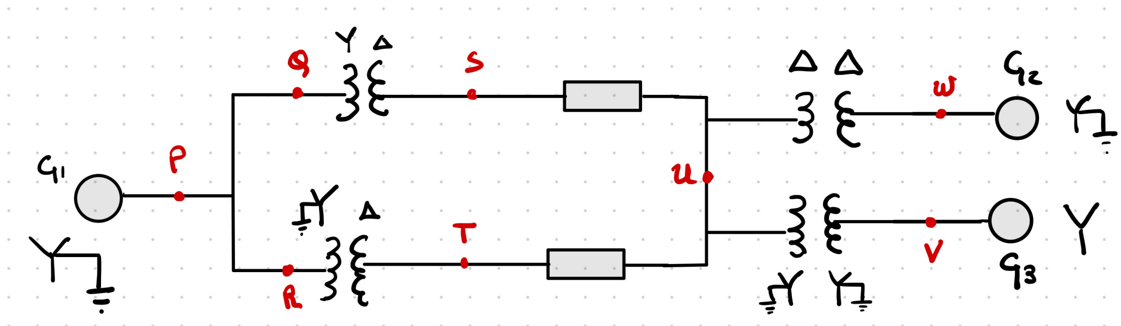

Figure 3: One-line diagram for the Problem 4 system.

Problem 4 (25pts)

Consider the one-line diagram shown in Fig. 3. The points P, Q, R, S, T, U, V, W label various points in this system. The positive-, negative-, and zero-sequence reactances for the generators are X_g^+, X_g^-, and X_g^0. Likewise, the transformers have positive, negative, and zero-sequence reances of X_T^+, X_T^-, and X_T^0, and the transmission lines (i.e., the lines connecting the points $S \rightarrow U$ and $T \rightarrow U$) have positive-, negative-, and zero-sequence reactances of X_l^+, X_l^-, and X_l^0.

Draw the positive-sequence, negative-sequence, and zero-sequence networks for the system in the one-line diagram. Label where the points P, Q, R, S, T, U, V, W, shown in Fig. 3, are equivalently located within your sequence networks.

Hints:

- *Note that the wye connection in the the transformer between points Q and S is not grounded.*
- *The generators G_1 and G_2 have a grounded wye connection, and G_3 is an ungrounded wye.*
- *Be sure to include the appropriate phase shifts for the transformers as needed.*