Agenda:

- Recapping material for Exam 2 → Adders
  → Decoders\Encoders
  → Multiplexers\Demultiplexers
- General complements
- sequential → Nonidealities

Exam: Oct 17th – Oct 19th

- Number systems
- Building blocks

Up next:

- State Machines,
- Latches
- Low-level

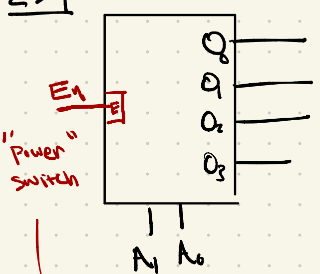
Exam guidelines:

- Please do not collaborate on the first pass
- Generative AI – let me know how you used it (if you did)
- You're responsible for the output

Recap: Decoders

→ Turns binary input into the index of an output signal in base 10

2→4



$O_i = 1 \Leftarrow En = 1$

For each i, $O_i = 1$ if $((A_1 A_0)_2)_{10} = i$

4 rows that we don't care about

En	A_1	A_0	O_3	O_2	O_1	O_0
0	X	X	0	0	0	0
1	0	0	0	0	0	1
1	0	1	0	0	1	0
1	1	0	0	1	0	0
1	1	1	1	0	0	0

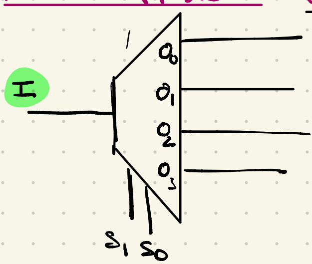
Predetermined to 1

$A_1 A_0$
$(1\ 0)_{10} = 2$

Enable signal

A_1	A_0	O_3	O_2	O_1	O_0
0	0				En
0	1			En	
1	0		En		
1	1	En			

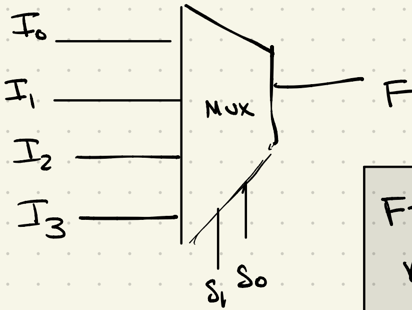
DeMultiplexers (e.g. "1→4")



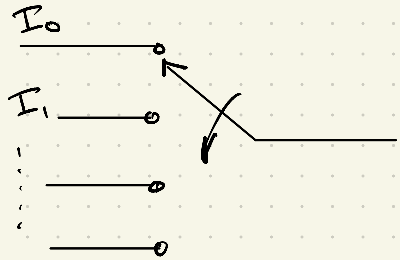
The enable is the input to DEMUX

Multiplexers – Selects one of many inputs

e.g. $4 \to 1$



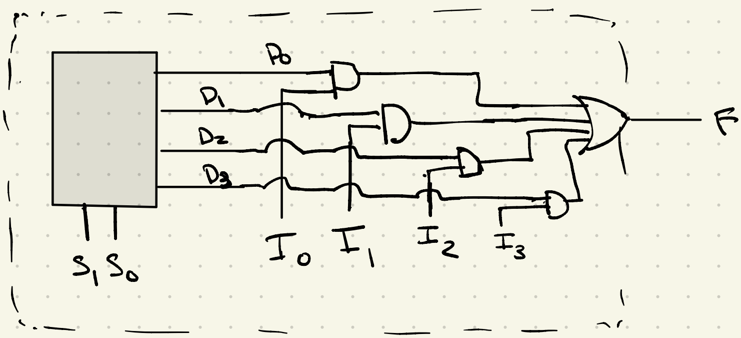
$\Longleftrightarrow$



$$\boxed{F = I_K \text{ when } K = \left((S_1 S_0)_2\right)_{10}}$$

S_1	S_0	I_3	I_2	I_1	I_0	F
0	0	0	0	0	I_0	I_0
0	1	0	0	I_1	0	I_1
1	0	0	I_2	0	0	I_2
1	1	I_3	0	0	0	I_3

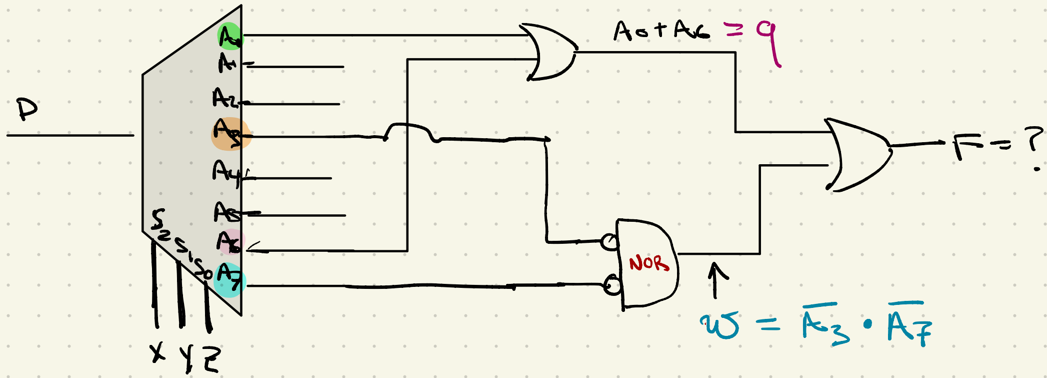
Relationship between Decoders & Multiplexers



$$F = D_0 I_0 + D_1 I_1 + D_2 I_2 + D_3 I_3$$

$$= \underbrace{\bar{S}_1 \bar{S}_0}_{\text{"00"}} I_0 + \underbrace{\bar{S}_1 S_0}_{\text{"01"}} I_1 + \underbrace{S_1 \bar{S}_0}_{\text{"10"}} I_2 + \underbrace{S_1 S_0}_{\text{"11"}} I_3$$

Example: $1 \to 8$ Demultiplexer



$A_0 = \bar{x} \cdot \bar{y} \cdot \bar{z}$ $\qquad$ $A_6 = x \cdot y \cdot \bar{z}$

$A_3 = \bar{x} \cdot y \cdot z$ $\qquad$ $A_7 = xyz$

$$F = w + q \triangleq \overline{A_3} \cdot \overline{A_7} + A_0 + A_6$$

$$= \overline{(\bar{x} \cdot y \cdot z)} \cdot \overline{(x \cdot y \cdot z)} + \bar{x} \cdot \bar{y} \cdot \bar{z} + x \cdot y \cdot \bar{z}$$

DeMorgan

$$= (x + \bar{y} + \bar{z}) \cdot (\bar{x} + \bar{y} + \bar{z}) + \bar{x} \cdot \bar{y} \cdot \bar{z} + x \cdot y \cdot \bar{z}$$

(FOIL)

$$= \underbrace{\cancel{x\bar{x}}}_{0} + x\bar{y} + \underbrace{x\bar{z}}_{\bar{z}} + \underbrace{\bar{y} \cdot \bar{x} + \bar{y}}_{\bar{y}} + \underbrace{\bar{z}}_{\bar{z}} + \bar{x}\bar{y}\bar{z} + xy\bar{z}$$

$$= \cancel{x\bar{y}} + \bar{z} + \cancel{\bar{y}} + \bar{x}\bar{y}\bar{z} + xy\bar{z}$$

$$= \bar{y} + \bar{z} + \bar{x}\bar{y} \cdot \bar{z} + xy\bar{z}$$

$$= \bar{y} + \bar{z}(1 + \cancel{\bar{x}\bar{y}}) + xy\bar{z}$$

$$= \bar{y} + \bar{z} + xy\bar{z}$$

$$\boxed{= \bar{y} + \bar{z}}$$

16's Complement Question (Challenge Question)

$$X = (D_{n-1} D_{n-2} \cdots D_1 D_0)_b = \sum_{K=0}^{n-1} D_K b^K$$

B length 4n (base 2) $\longleftrightarrow$ H

ex $B = 1011\ 0011$ $\quad n = 2$ $\quad B = (\beta_{n-1} \beta_{n-2} \cdots \beta_0)$

$\underbrace{1011}\ \underbrace{0011}$

$(B\ \ 3)_{16}$

Hex # $= (11)_{10}$

$$B = \sum_{K=0}^{4n-1} \beta_i 2^i = \sum_{K=0}^{n-1} h_i \cdot 16^i$$

Ones' Complement of B

$$-B = 1^{4n} - \sum_{K=0}^{4n-1} \beta_K 2^K$$