Georgia Tech College of Engineering
School of Electrical
and Computer Engineering

ECE 2020 Lecture 1: Digital logic signals

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Logistics

- Office hours starting next week TR/F 11:30am-1:00pm, Van Leer C248
- **Course survey:** fill out to let me know your feedback on...
  - Lecture style preferences
  - Content delivery
  - Homework flexibility preferences
- First homework assignment posted (just to get a taste).
  1. Want you to have a peaceful labor day
  2. Want your feedback early, so I can make this course more fun and helpful.
  3. If the workload is too high, **please communicate this**, don't suffer in silence.

Agenda

- Manipulating digital signals with Boolean logic
- Rules for Boolean operations
- Converting digital signals to different forms:
    - Truth tables
    - Boolean algebraic form
    - Gate-level schematics
    - Switching logic
- Understanding switching operations

A few words

My goals this semester:

- I want to **learn from you**.
- I want to give you some interest in Electrical Engineering.
- I want everyone to succeed.
- I want everyone to feel welcome and comfortable.

How to succeed

Here's how to succeed in this course:

- This is an introductory course. **There is no assumed knowledge in this course.** If you don't understand something, **stop me**.
- Come to office hours so that we can learn from eachother **(starts next week!)**.
- Use Piazza freely. There are no stupid questions.

Recap

Digital signal

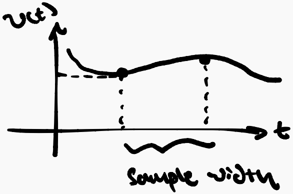
A digital signal, or **binary**, **logic**, or **Boolean** signal $\underline{X}$ is a variable that can take the values **one** or **zero** depending on whether some other thing is **true** or **false**, i.e.:

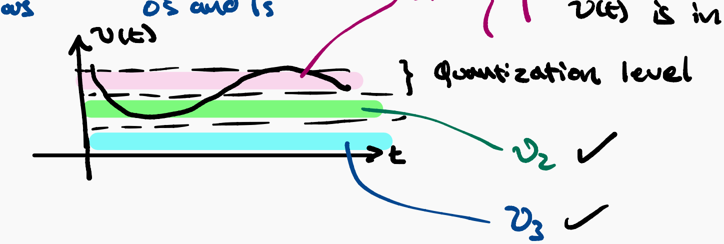
$$X = \begin{cases} 1 & \text{if some thing is true} \\ 0 & \text{if some thing is false.} \end{cases} \tag{1}$$

Digital signal

Transforming analog → digital

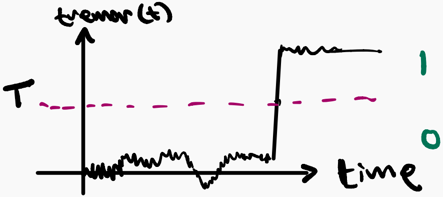
"Continous" "0's and 1's"





✓ $v_1 = \begin{cases} 0 & v(t) \text{ not in bucket 1} \\ 1 & v(t) \text{ is in bucket 1} \end{cases}$

Earthquake Detection



$Z = \begin{cases} 1 & \text{if tremor} > T \\ 0 & \text{otherwise} \end{cases}$

Logic gates = Building blocks

What are logic gates?

Logic gates are **the basic building blocks of digital circuits**. They allow you to encode **human decision-making** in an electrical circuit.

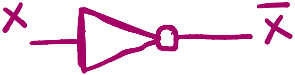
Logic gates let us **avoid thinking about electricity**, and instead focus on the **problem we want to solve**.

Logic gates

NOT

X	$Z = \overline{X}$
0	1
1	0

Table 1: NOT gates: “swap please!”

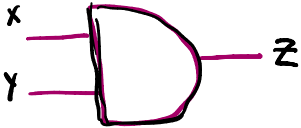


Logic gates

AND

Input		Output
X	Y	$Z = X \cdot Y$
0	0	0
0	1	0
1	0	0
1	1	1

Table 2: AND gates: "gotta have it all!"



A	B	C	Z
0	0	0	
0	0	1	
0	1	0	
1	0	0	
0	1	1	
1	1	0	
1	0	1	
1	1	1	

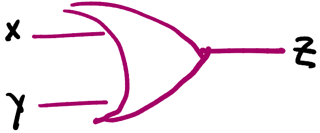
$$N_{inputs} = 2^{N_{signals}}$$

Logic gates

OR

X	Y	$Z = X + Y$
0	0	0
0	1	1
1	0	1
1	1	1

Table 3: OR gates: “either is fine!”

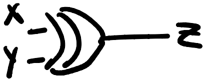


Logic gates

XOR

X	Y	$Z = X \oplus Y$
0	0	0
0	1	1
1	0	1
1	1	0

Table 4: XOR: “one or the other, please!”



Boolean algebra

What is Boolean Algebra?

Boolean algebra is a set of **mathematical rules** for manipulating digital signals.

It is useful for:

(1) Simplifying gate-level logic circuits

(2) Understanding how a system behaves

(3) Creating new inputs or outputs to other systemsto other systems

Order of operations

The **order of operations** for Boolean Algebra is *almost* the same as normal algebra:

(1) Parentheses

(2) NOT

(3) AND

(4) OR

Boolean identities

The Boolean identities are a series of laws that describe how to manipulate digital signals. We will go through them now.

1.) "Identity" $\underbrace{X+0=X}_{\text{"Identity"}}$, $\underbrace{X\cdot 1=X}_{\text{"Identity"}}$

2.) $X+X=X$, $X\cdot X=X$ "Idempotence"

3.) Complement

$$X+\overline{X}=1 \qquad X\cdot\overline{X}=0$$

4.) Involution "nots cancel" $\overline{\overline{X}}=X$

5.) Commutative: $X+Y=Y+X$, $XY=YX$

6.) Associative: $X+(Y+Z)=(X+Y)+Z$

$X(YZ)=(XY)Z$

7.) Distributive: $X(Y+Z)=XY+XZ$

8.) Absorption: $X+XY=X$, $X(X+Y)=X$

9.) Simplification: $X+\overline{X}Y=X+Y$

10.) $\boxed{\text{De Morgan's Theorem: } \overline{X+Y}=\overline{X}\cdot\overline{Y}}$ $\Longleftrightarrow$ $\overline{X\cdot Y}=\overline{X}+\overline{Y}$

Examples

Example 1

Let's consider a friendly logic function:

$$F(A, B, C) = C + \overbrace{(\underbrace{\overline{C}}_{=0}B + B\underbrace{\overline{A}}_{0})}^{=0} . = 1$$

What is the value of of

$$F(\overset{A}{1}, \overset{B}{0}, \overset{C}{1}) = ...?$$

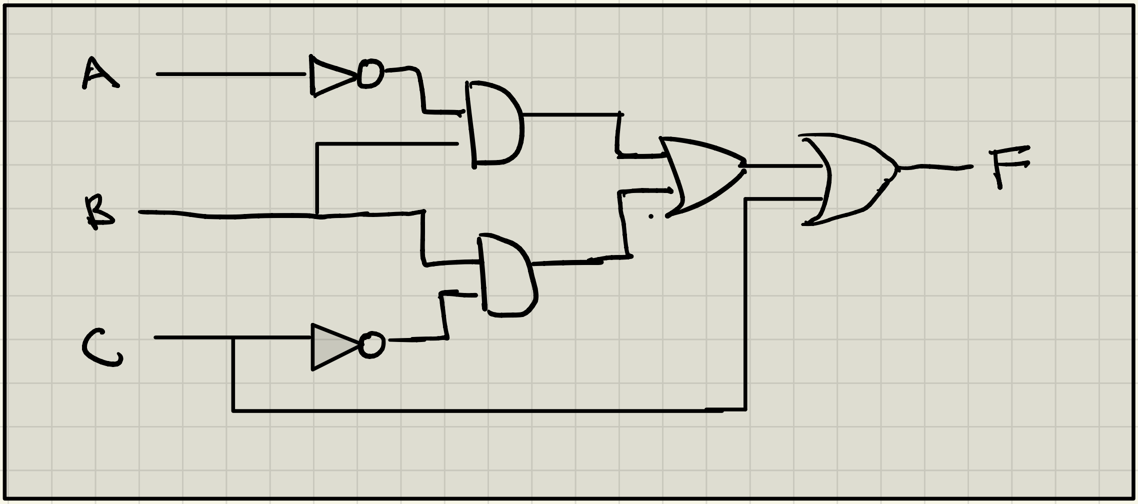
Draw the logic gate circuit.

$$F(A,B,C) = C + (\overline{C}B + B\overline{A})$$

$$F(1,0,1) = 1 + (\underbrace{\overline{1}\cdot 0}_{=0} + \underbrace{0\cdot\overline{1}}_{=0})$$

$$= 1. \quad \therefore \boxed{F(1,0,1)=1}$$

Gate-Level Schematic



Examples

Example 2

Let's consider a slightly less friendly logic function:

$$F(A, B, C, D) = A \cdot \overline{\left(B + \overline{C} \cdot D\right)} + \overline{A} \cdot \overline{\left(\overline{B} + C\right)}.$$

What is the value of

$$F(1, 0, 0, 1) = \text{...?}$$

Draw the logic gate circuit.

Examples

$0 \cdot X = 0$!

Example 2

Let’s consider a slightly less friendly logic function:

$$F(A,B,C,D) = \underbrace{A}_{1} \cdot \underbrace{\overline{(\underbrace{\underbrace{B}_{0} + \underbrace{\overline{C}}_{1}}_{1} \cdot \underbrace{D}_{1})}}_{(1\cdot 1)=0} + \underbrace{\overset{0\cdot}{\overline{A}} \cdot \overline{(\overline{B}+C)}}_{=0}.$$

What is the value of

$$F\underbrace{(1,0,0,1)}_{A,B,C,D} = \underline{0}?$$

Draw the logic gate circuit.

$$F(A,B,C,D) = A\cdot\overline{(B+\overline{C}\cdot D)} + \overline{A}\cdot\overline{(\overline{B}+C)}$$

Q: What is $F(1,0,0,1) = ?$

$$F(1,0,0,1) = 1\cdot\overline{(0+\underbrace{\underbrace{\overline{0}}_{=1}\cdot 1}_{=1})} + \underbrace{\underbrace{\overline{1}}_{=0}\cdot\overline{(\overline{0}+0)}}_{=0}$$

$$= 1\cdot\overline{1} + 0$$

$$= 0. \quad \therefore \quad \boxed{F(1,0,0,1)=0.}$$

Gate-Level Schematic