Digital System Design, Exam 1 (Fall 2024)

ECE 2020-IE
09/19/2024

Name: _____

I, _____, have neither given nor received unauthorized help on this exam, and I have conducted myself within the guidelines of the honor code of the Georgia Institute of Technology.

Please read this information:

- Please show all your work.
- Please do not access the internet or any other communications channel.
- Please box or circle your final answers.
- This test has 3 problems that total up to 100 points.
- This test has 1 bonus problem worth up to 20 points.
- You have until the end of class to complete.

Boolean Identities

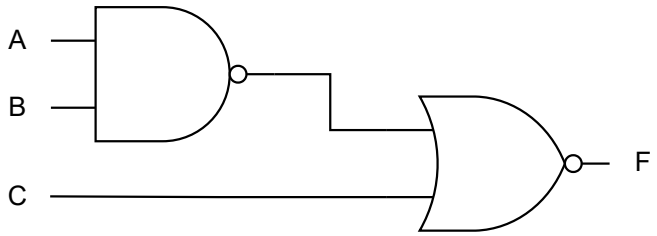
- Identity:
  - $A + 0 = A$
  - $A \cdot 1 = A$
- Dominance:
  - $A + 1 = 1$
  - $A \cdot 0 = 0$
- Idempotence:
  - $A + A = A$
  - $A \cdot A = A$
- Inverse:
  - $A + \overline{A} = 1$
  - $A \cdot \overline{A} = 0$
- Commutative:
  - $A + B = B + A$
  - $A \cdot B = B \cdot A$
- Associative:
  - $A + (B + C) = (A + B) + C$
  - $A \cdot (B \cdot C) = (A \cdot B) \cdot C$
- Distributive:
  - $A \cdot (B + C) = A \cdot B + A \cdot C$
  - $A + B \cdot C = (A + B) \cdot (A + C)$
- Absorption:
  - $A \cdot (A + B) = A$
  - $A + A \cdot B = A$
- DeMorgan's:
  - $\overline{(A + B)} = \overline{A} \cdot \overline{B}$
  - $\overline{(A \cdot B)} = \overline{A} + \overline{B}$
- Double Complement:
  - $\overline{\overline{A}} = A$
- FOIL:
  - $(A + B) \cdot (C + D) = A \cdot C + A \cdot D + B \cdot C + B \cdot D$
- Disappearing Opposite:
  - $A + \overline{A} \cdot B = A + B$

Problem 1: Logic Gates (30pts)

Question 1a. (10 pts)

Write a Boolean Algebra expression for the output of the logic circuit shown below.

$F =$

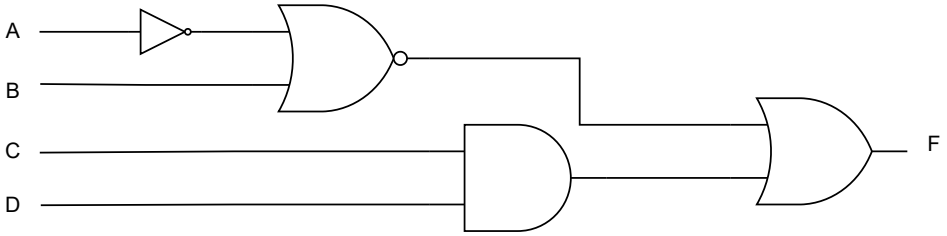


Logic Circuit 1.a.

Question 1b. (10 pts)

Write a Boolean Algebra expression for the output of the logic circuit shown below.

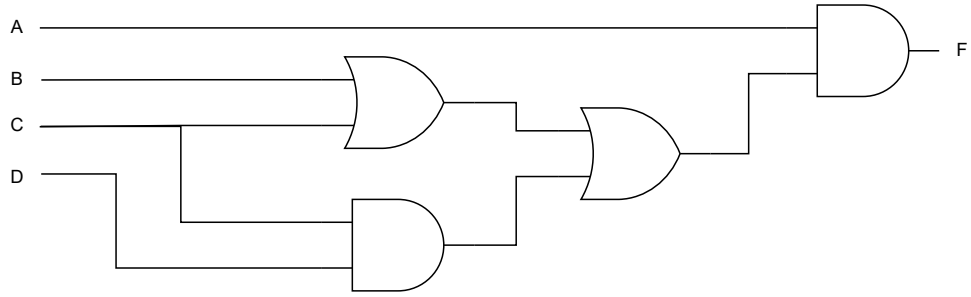
$F =$



Logic Circuit 1.b.

Question 1d. (10 pts)

Draw the switching logic for the logic circuit shown below, without simplifying anything.
Hint: Only normally open switches are needed for this problem.



Logic Circuit 1.d.

Problem 2: CMOS (30pts)

Consider the following logic function:

$$F = \overline{\overline{A \cdot B} + C}. \tag{1}$$

Question 2a. (15 pts)

Derive expressions for *pull-up* and *pull-down* networks that implement F and $\overline{F}$.

pull-up =

pull-down =

Question 2b. (15 pts)

Draw a schematic for the CMOS implementation of the logic function F. Label the locations of the source voltage v_{dd} and output voltage v_{out}. Indicate the pull-up network and the pull-down network.

Note: You can use inputs like $\overline{A}$ directly, without an inverter.
*Hint: **N**MOS transistors are used in pull-down networks and **P**MOS transistors are used in pull-up networks.*

Problem 3: Boolean Algebra (30pts)

Question 3a. (15 pts)

Using Boolean Algebra identities, simplify the following logic function. Justification earns partial credit.

$$F = A \cdot B \cdot C + \overline{A} + A \cdot \overline{B} \cdot C \tag{2}$$

Question 3b. (15 pts)

Using Boolean Algebra identities, simplify the following logic function. Justification earns partial credit.

$$F = \overline{A \cdot (\overline{B} \cdot \overline{C} + B \cdot C)} \tag{3}$$

Question (Bonus). (20 pts)

Let X, Y, Z be Boolean variables. Consider the following logic function:

$$F = X \cdot Y + Z \cdot Y + \overline{X} \cdot Z. \tag{4}$$

It is possible to simplify F to:

$$F_{\mathsf{new}} = X \cdot Y + \overline{X} \cdot Z. \tag{5}$$

This is known as the *Consensus Theorem*. Use the empty truth table and K-map shown below to demonstrate that the Consensus Theorem is true, by showing that $F = F_{\mathsf{new}}$.

Hint: As long as all 1s remain covered by other groups, you can remove any single grouping from a K-map.
Note: The only column you need to fill in the truth table is F, all other columns are optional.

X	Y	Z	$X \cdot Y$	$Z \cdot Y$	$\overline{X} \cdot Z$	F
0	0	0				
0	0	1				
0	1	0				
0	1	1				
1	0	0				
1	0	1				
1	1	0				
1	1	1				

		YZ			
		00	01	11	10
X	0				
	1				

Scratch paper; if used, please clearly indicate which question you are working on