ECE 2020 IE3, Fall 2024

# Problem Set #1: Solutions

Due: August 30, 2024

3 problems, 20 points

## Smart Thermostat

You want to design a smart thermostat that intelligently manages whether an HVAC unit is heating or cooling to improve the efficiency of a home.

### Inputs

You have access to the switch to the power supply of an HVAC unit $X$, which you can write as

$$X = \begin{cases} 0 & \text{when power of HVAC (heating or cooling) is off} \\ 1 & \text{when power of HVAC (heating or cooling) is on.} \end{cases} \tag{1}$$

When the thermostat reaches a comfort temperature, it reports this through the logical signal $T$. You can write this signal as

$$T = \begin{cases} 0 & \text{when temperature} < \text{comfort}^\circ \\ 1 & \text{otherwise.} \end{cases} \tag{2}$$

You want to design the system to only heat or cool to the comfort temperature when motion is detected, which is reported by a motion sensor's logic signal $M$,

$$M = \begin{cases} 0 & \text{last motion detected} > 30\text{min ago} \\ 1 & \text{last motion detected} < 30\text{min ago.} \end{cases} \tag{3}$$

### Outputs

Now, you need your thermostat to report whether the HVAC is operating in heating or cooling mode. You define two new logic signals, $C$ and $H$, as your *outputs* which you write as

$$C = \begin{cases} 1 & \text{set HVAC in cooler mode} \\ 0 & \text{otherwise,} \end{cases} \qquad H = \begin{cases} 1 & \text{set HVAC in heater mode} \\ 0 & \text{otherwise.} \end{cases}$$

**Problem 1** ($2 \times 4 = 8$ points possible)

1. Derive logic functions that implement the cooling and heating output signals $C$ and $H$ for your Smart Thermostat.
2. Create a truth table for the system. Use signals $X, T, M$ as input and signals $C, H$ as output. Indicate the number of possible inputs for this system (i.e., the number of rows of the truth table).
3. Draw gate-level schematics for the output signals $C$ and $H$.
4. Define a new output signal

$$Y = \begin{cases} 1 & \text{HVAC is operating in either cooler or heater mode} \\ 0 & \text{HVAC is not operating.} \end{cases}$$

Derive the simplest possible expression for $Y$.

# Problem 1 Solution:

## Part 1

We want to heat the house when the temperature falls *below* the comfort temperature, i.e. $\overline{T} = 1$. So, we conclude

$$H = X \cdot M \cdot \overline{T}, \qquad C = X \cdot M \cdot T.$$

## Part 2

There are $2^3 = 8$ possible inputs. The truth table takes the following form.

| $X$ | $T$ | $M$ | $\overline{T}$ | $C$ | $H$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 | 0 |

## Part 3

Lots of variants of the logic gate diagrams are accepted.

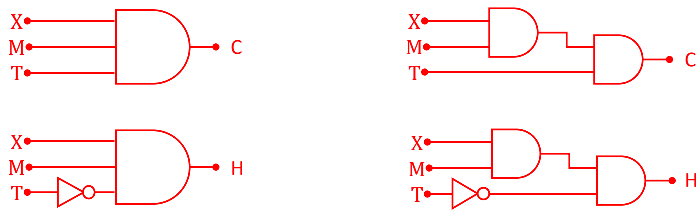


Figure 1: The smart thermostat system

**Part 4**

We have

$$\begin{aligned} Y &= C + H \\ &= XM(T + \overline{T}) \\ &= XM \cdot 1 = XM, \end{aligned}$$

where we used $T + \overline{T} = 1$.

## Fun With Schematics

**Problem 2** ($2 \times 4 = 8$ points possible)
Consider the following two logic functions. Draw both

1. gate-level schematics, and
2. switch logic schematics,

without simplifying the expressions. Do not use complemented variables, e.g., $\overline{A}$ as inputs; instead, draw the NOT gates or normally-closed switches in the schematic.

$$F_1 = (A \cdot B) + C \cdot (A + B) + C \cdot A + B \tag{4a}$$
$$F_2 = \overline{A} + AB + \overline{C}D\overline{A} + \overline{C}D \tag{4b}$$

## Problem 2 solution:

### $F_1$ schematics

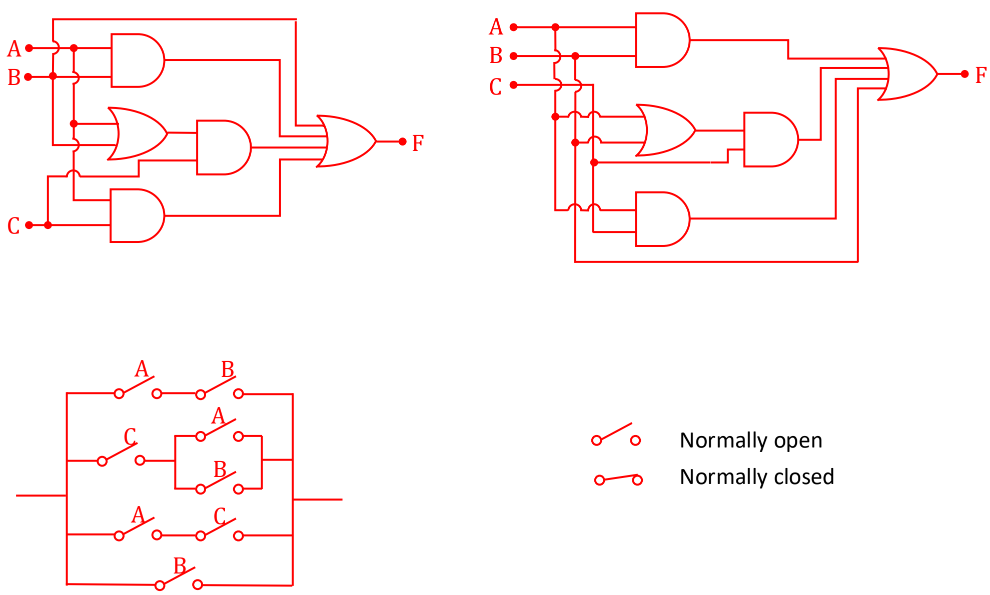


Figure 2: The $F_1$ switching and logic schematics

## $F_2$ schematics

$$F = \bar{A} + AB + \bar{C}D\bar{A} + \bar{C}D$$

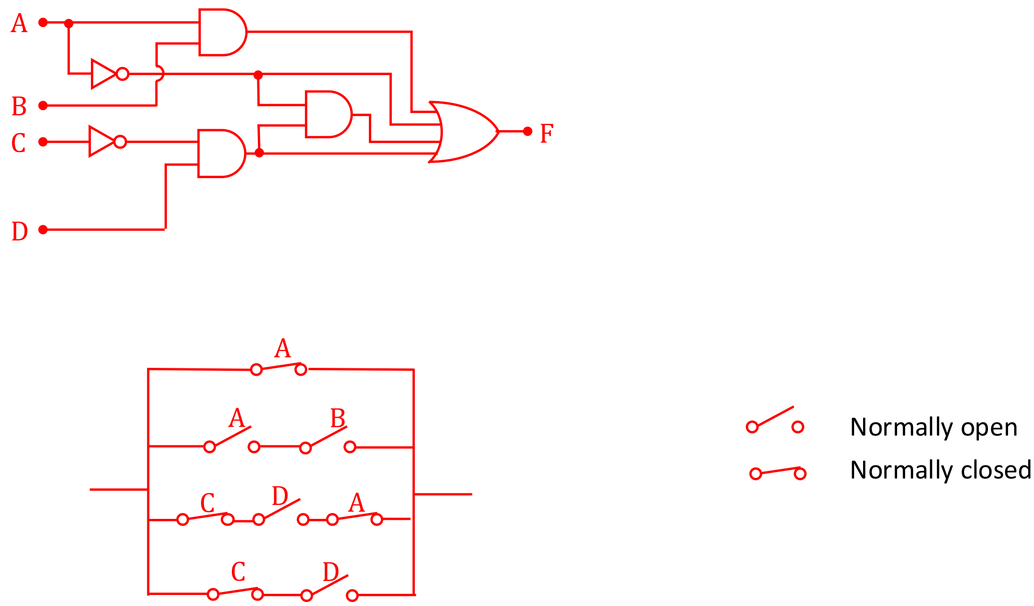


Figure 3: The $F_2$ switching and logic schematics

## Boolean Algebra

Study the *Boolean Identities* document on Canvas.

**Problem 3** ($2 \times 2 = 4$ points possible)
Derive the simplest possible forms of the following logic functions, providing each step of your derivation. *No need to draw the circuits.*

$$F_1 = (A \cdot B) + C \cdot (A + B) + C \cdot A + B \tag{5a}$$
$$F_2 = \overline{A} + AB + \overline{C}D\overline{A} + \overline{C}D \tag{5b}$$

## Problem 3 Solution:

$F_1$

We have that

$$\begin{aligned} F_1 &= (A \cdot B) + C \cdot (A + B) + C \cdot A + B \\ &= AB + CA + CB + CA + B \\ &\overset{(1)}{=} AB + CA + CB + B \\ &\overset{(2)}{=} AB + CA + B \\ &\overset{(3)}{=} B + CA. \end{aligned}$$

where step (1) is by noting that $CA + CA = CA$, step (2) is by absorption: $CB + B = B$, and step (3) is by absorption again: $AB + B = B$.

$F_2$

We obtain

$$\begin{aligned} F_2 &= \overline{A} + AB + \overline{C}D\overline{A} + \overline{C}D \\ &= \overline{A} + AB + \overline{C}D(\overline{A} + 1) \\ &= \overline{A} + AB + \overline{C}D \\ &\overset{(1)}{=} \overline{A}\underbrace{(B+1)}_{=1} + AB + \overline{C}D \\ &\overset{(2)}{=} \overline{A} + B\underbrace{(\overline{A}+A)}_{=1} + \overline{C}D \\ &\overset{(3)}{=} \overline{A} + B + \overline{C}D \end{aligned}$$

where in step (1) we introduce an AND with $(B + 1)$ without changing anything, and in step (2) we apply the distributive property and factor. The result in step (3) is known as the *disappearing opposite* theorem: $\overline{A} + AB = \overline{A} + B$., or equivalently, $A + \overline{A}B = A + B$.