Smart Thermostat

You want to design a smart thermostat that intelligently manages whether an HVAC unit is heating or cooling to improve the efficiency of a home.

Inputs

You have access to the switch to the power supply of an HVAC unit X, which you can write as

$$X = \begin{cases} 0 & \text{when power of HVAC (heating or cooling) is off} \\ 1 & \text{when power of HVAC (heating or cooling) is on.} \end{cases} \tag{1}$$

When the thermostat reaches a comfort temperature, it reports this through the logical signal T. You can write this signal as

$$T = \begin{cases} 0 & \text{when temperature} < \text{comfort}^\circ \\ 1 & \text{otherwise.} \end{cases} \tag{2}$$

You want to design the system to only heat or cool to the comfort temperature when motion is detected, which is reported by a motion sensor's logic signal M,

$$M = \begin{cases} 0 & \text{last motion detected} > 30\text{min ago} \\ 1 & \text{last motion detected} < 30\text{min ago.} \end{cases} \tag{3}$$

Outputs

Now, you need your thermostat to report whether the HVAC is operating in heating or cooling mode. You define two new logic signals, C and H, as your *outputs* which you write as

$$C = \begin{cases} 1 & \text{set HVAC in cooler mode} \\ 0 & \text{otherwise,} \end{cases} \qquad H = \begin{cases} 1 & \text{set HVAC in heater mode} \\ 0 & \text{otherwise.} \end{cases}$$

Problem 1 ($2 \times 4 = 8$ points possible)

1. Derive logic functions that implement the cooling and heating output signals C and H for your Smart Thermostat.
2. Create a truth table for the system. Use signals X, T, M as input and signals C, H as output. Indicate the number of possible inputs for this system (i.e., the number of rows of the truth table).
3. Draw gate-level schematics for the output signals C and H.
4. Define a new output signal

$$Y = \begin{cases} 1 & \text{HVAC is operating in either cooler or heater mode} \\ 0 & \text{HVAC is not operating.} \end{cases}$$

   Derive the simplest possible expression for Y.

Fun With Schematics

Problem 2 ($2 \times 4 = 8$ points possible)
Consider the following two logic functions. Draw both

1. gate-level schematics, and
2. switch logic schematics,

without simplifying the expressions. Do not use complemented variables, e.g., $\overline{A}$ as inputs; instead, draw the NOT gates or normally-closed switches in the schematic.

$$F_1 = (A \cdot B) + C \cdot (A + B) + C \cdot A + B \tag{4a}$$

$$F_2 = \overline{A} + AB + \overline{C}D\overline{A} + \overline{C}D \tag{4b}$$

Boolean Algebra

Study the *Boolean Identities* document on Canvas.

Problem 3 ($2 \times 2 = 4$ points possible)

Derive the simplest possible forms of the following logic functions, providing each step of your derivation. *No need to draw the circuits.*

$$F_1 = (A \cdot B) + C \cdot (A + B) + C \cdot A + B \tag{5a}$$

$$F_2 = \overline{A} + AB + \overline{C}D\overline{A} + \overline{C}D \tag{5b}$$