Georgia Tech College of Engineering
School of Electrical and Computer Engineering

Strategic distribution network sensing

Samuel Talkington, Rahul Gupta, Richard Asiamah, Paprapee Buason, Daniel K Molzahn

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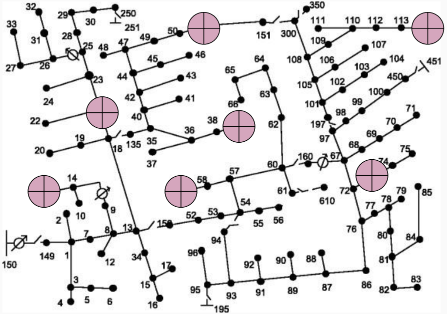
Traditional sensor placement: offline, static

With sensors



Without sensors

Select *locations* (a subset of nodes) to install sensors and measure continuously

Motivation

Dave Rieken,
Vice President of
Research

HUBBELL

Sensor *placement* in power systems is **wrong**!

We have sensors at ***every* node:** The problem is ***how* we sense!**

We make smart meters, folks!

70% of North American households have smart meters, EIA, 2020.

Here's an algorithm that selects sensor locations

Samuel Talkington

Sensor placement–or sampling?

Dave Rieken,
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Research

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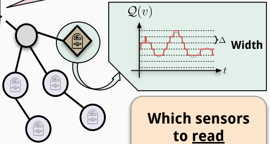
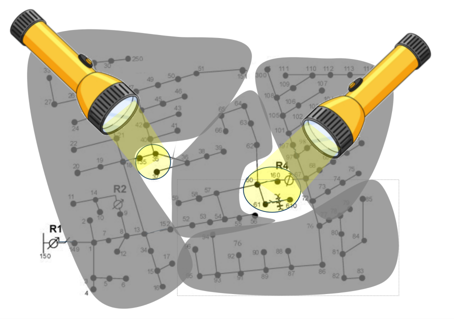
The key limitation is a
communication
bandwidth limit



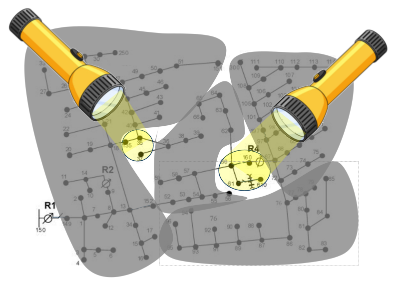
Which sensors
to read



70% of North American households have smart meters, EIA, 2020.

The problem

- Power distribution networks have high levels of sensors already, but with...
- *Limited communication* ***bandwidth***.
- **How do we dynamically monitor these sensor networks efficiently?**
- **i.e., how to move these flashlights around?**



Power distribution systems

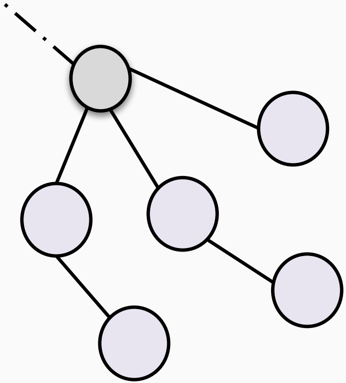


Figure 1: A distribution network can be modeled as a **tree network**, $|\mathcal{N}| = n$, and $|\mathcal{E}| = n - 1$.

Select only a few sensors

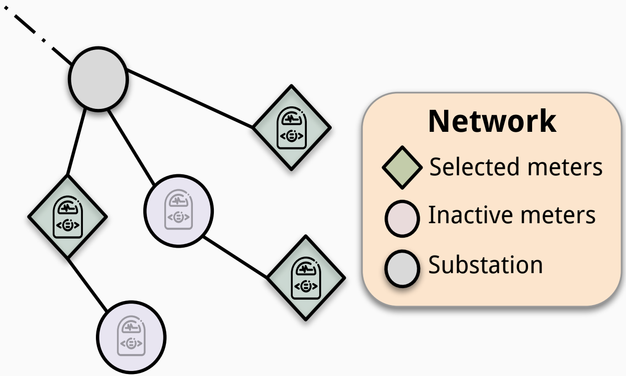


Figure 2: Key idea: we can only select a few sensors

Select $\mathcal{S}$, find worst case in $\mathcal{S}$

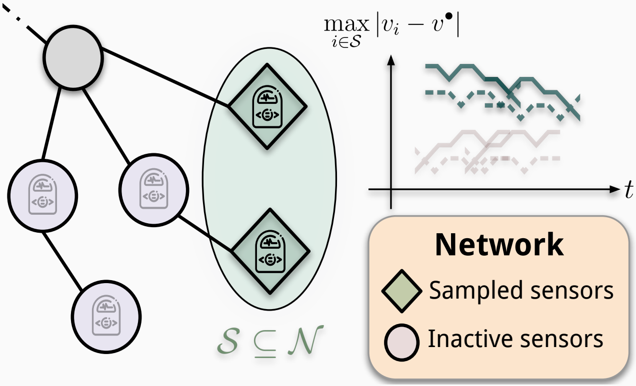


Figure 3: From $\mathcal{S}$, what's the **worst case** voltage?

Grid model

Power flow equations: Recap

- A grid is a graph: $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, with $n = |\mathcal{N}|$ nodes.
- Nodal voltages: $\boldsymbol{u} = \boldsymbol{v} \circ \exp(j\boldsymbol{\theta}) \in \mathbb{C}^n$

  $\boldsymbol{v} \in \mathbb{R}^n$ voltage magnitudes

  $\boldsymbol{\theta} \in (-\pi, \pi]^n$ voltage phase angles
- Nodal power injections: $\boldsymbol{s} = \boldsymbol{p} + j\boldsymbol{q} \in \mathbb{C}^n$

  $\boldsymbol{p} \in \mathbb{R}^n$, "active" power

  $\boldsymbol{q} \in \mathbb{R}^n$, "reactive" power
- $\boldsymbol{Y} \in \mathbb{C}^{n \times n}$ nodal admittance matrix (generalized, complex-valued graph Laplacian)

Power flow equations $\boldsymbol{s} : \mathbb{C}^n \to \mathbb{C}^n$

$$\boldsymbol{s} = \text{diag}(\boldsymbol{u}) \underline{\boldsymbol{Y} \boldsymbol{u}}$$

Linear power flow model

A simple power flow model is formed by inverting the power flow Jacobian at the flat start condition:

$$\begin{bmatrix} p \\ q \end{bmatrix} \approx \begin{bmatrix} G & -B \\ -B & -G \end{bmatrix} \begin{bmatrix} v - 1 \\ \theta \end{bmatrix} \iff \begin{bmatrix} v - 1 \\ \theta \end{bmatrix} \approx \begin{bmatrix} R & X \\ X & -R \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix}, \qquad (1)$$

where $G, B \succeq 0$ are the real and imaginary components of the $n \times n$ reduced admittance matrix $Y = G + \mathrm{j}B$, and $R, X \succeq 0$ are the *resistance and reactance* matrices.

Linear power flow model

For *distribution (tree) networks*, the voltage magnitudes $\boldsymbol{v} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ can be approximated as a linear system:

$$\boldsymbol{v} \approx \mathbf{1} + \boldsymbol{R}\boldsymbol{p} + \boldsymbol{X}\boldsymbol{q}.$$

Linear distribution network model

Denoting $\boldsymbol{\epsilon} := \boldsymbol{v} - \mathbf{1}$ as the *voltage magnitude perturbations*, we will analyze:

$$\boxed{\boldsymbol{\epsilon} = \boldsymbol{R}\boldsymbol{p} + \boldsymbol{X}\boldsymbol{q}.}$$

Uncertain linear power flow model

Uncertainty in p, controllable q

Assumptions

Introduce generic uncertainty with the following assumptions:

- The reactive power injections $\boldsymbol{q}$ are set by a linear controller with a gain (ratio of reactive to active injections): $\kappa = q_i / p_i$, that is known for all nodes.
- The active power injections $\boldsymbol{p}$ are **random** with an **unknown distribution** with bounds $p_i \in \left[\underline{p}, \overline{p}\right]$ computed from historical data.

Key points about randomness in p:

- The uncertainty assumptions for $\boldsymbol{p}$ that are **neither Gaussian, independent, nor identically distributed.**
- Only requires **bounds**, which can be arise in engineering contexts such as:
  - Hosting capacity values.
  - Global horizontal irradiance (GHI) clear sky model data.
  - Device manufacturer limits.
  - Optimal power flow or other engineering constraints.

Main result

Theorem (Concentration of Voltages Under Uncertain Power Injections)

Let $\mathbf{p}$ be an n-dimensional vector of random active power injections that are bounded between $\overline{p}$ and $\underline{p}$, and let $\Delta := \overline{p} - \underline{p}$ denote the bound width. Let $\boldsymbol{K}$ be a fixed $n \times n$ control matrix such that $\mathbf{q} = \boldsymbol{K}\mathbf{p}$. Then $\mathbf{v} = \mathbf{1} + (\boldsymbol{R} + \boldsymbol{X}\boldsymbol{K})\,\mathbf{p}$, and perturbations in nodal voltages satisfy

$$\mathrm{E}\left[||\mathbf{v} - \mathbf{1}||_\infty\right] \leq \frac{1}{2}\Delta\,||\boldsymbol{R} + \boldsymbol{X}\boldsymbol{K}||_\infty \sqrt{2\log(2n)}; \tag{2}$$

moreover, for any $t > 0$,

$$\Pr\left[||\mathbf{v} - \mathbf{1}||_\infty > t\right] \leq 2n\exp\left\{\frac{-2t^2}{\Delta^2\,||\boldsymbol{R} + \boldsymbol{X}\boldsymbol{K}||_\infty^2}\right\}. \tag{3}$$

Graph Fourier transform

From the fixed power factor assumption, there is an orthonormal $\boldsymbol{W} \in \mathbb{R}^{n \times n}$, specifically, a **graph Fourier basis**, such that $\psi := \boldsymbol{W}^\top \epsilon$ is the *graph Fourier transform* of the voltage magnitudes. In summary,

$$\epsilon = \underbrace{(\boldsymbol{R} + \boldsymbol{X}\boldsymbol{K})}_{=\boldsymbol{L}^{-1}} \boldsymbol{p} = \boldsymbol{W}\boldsymbol{\Lambda}^{-1}\boldsymbol{W}^\top \boldsymbol{p} = \boldsymbol{W}\psi \tag{4}$$

Benefit: There exist efficient algorithms for sampling sensors with this special structure (more on this later).

Spectral bandit algorithm outline

Strategy

At each time t: the learner picks b nodes to **check the security.**

The set of all strategies is the **set of all subsets of b nodes.**

$$\mathcal{A} = \left\{ \mathcal{S} \in 2^{\mathcal{N}} \ : \ |\mathcal{S}| \le b \right\}, \tag{5}$$

so there are $|\mathcal{A}| = \binom{n}{b}$ possible strategies… challenging in general!

Reward

When the learner has selected sensors $\mathcal{S}_t \in \mathcal{A}$ to *ping*, she observes a *reward* $f : \mathcal{A} \to \mathbb{R}$ that looks like

$$\boxed{f(\mathcal{S}) = \text{Worst case voltage in } \mathcal{S}.} \tag{6}$$

In symbols:

$$f(\mathcal{S}) = \max_{i \in \mathcal{S}_t} |\epsilon_i| = \max_{i \in \mathcal{S}_t} |v_i - 1| = \max_{i \in \mathcal{S}_t} |\langle \mathbf{w}_i, \psi \rangle| \,. \tag{7}$$

This reward is the **maximum voltage magnitude observed in the sampling strategy.**

How to catch a bandit

To pick the best sampling strategy, minimize the **regret**:

$$\boxed{\text{Regret} = \text{E}\left[\text{Best voltage sampling strategy} - \text{Your voltage sampling strategy}\right]}$$

If at first you don't succeed... try again!

Spectral bandit algorithm

Solution approach: At each timestep t, recursively compute an estimate of the *Fourier coefficients* ψ for the voltage magnitudes $\boldsymbol{v}$:

$$\hat{\psi}_t = \underset{\boldsymbol{\psi} \in \mathbb{R}^n}{\arg\min} \sum_{s=1}^{t-1} \left(v_s - \langle \boldsymbol{w}_s, \boldsymbol{\psi} \rangle \right)^2 + \beta \, ||\boldsymbol{\psi}||_{\boldsymbol{\Lambda}}^2 , \tag{8}$$

where $\beta > 0$ is a regularization parameter that you choose. The indices $s = 1, \ldots t-1$ are the **sampled nodes!**

Spectral regularization

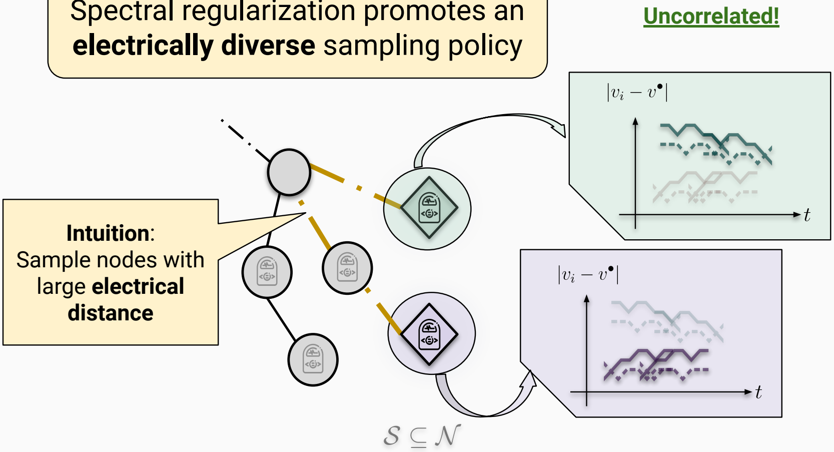
The regularization term, $||\psi||_{\mathbf{\Lambda}}$, promotes predictions of the voltages that are **electrically diverse**:

$$||\psi||_{\mathbf{\Lambda}} := \sqrt{\psi^{\top}\boldsymbol{L}\psi} = \sqrt{\sum_{(i,j)\in\mathcal{E}} y_{ij}(\psi_i - \psi_j)^2}. \tag{9}$$

This is also known as the **Dirichlet energy** of the graph.

Relates to *effective resistance*...check out the paper for more information

Spectral regularization promotes an **electrically diverse** sampling policy



Bandit algorithm solution

The regression problem has a closed form solution at each timestep t:

$$\hat{\psi}_t = \left(\sum_{s=1}^{t-1} \mathbf{w}_s \mathbf{w}_s^\top + \beta \mathbf{\Lambda} \right)^{-1} \left(\sum_{s=1}^{t-1} \mathbf{w}_s v_s \right) := \mathbf{V}_t^{-1} \left(\sum_{s=1}^{t-1} \mathbf{w}_s v_s \right).$$

Where $s = 1, \ldots, t-1 \in \mathcal{N}$ are the **sampled nodes!** The voltage at one node is often similar to its neighbor.

Q: How do we pick those samples?

Answer: Need to **bridge the gap** between the signal processing technique (spectral bandits) and the structural concentration results.

How do we pick those sampled nodes?

Selecting the sample s for each time step:

- Given a sampling budget b, pick the top b nodes ranked by upper confidence bounds on the voltages
- Estimate $\hat{\psi}_t$
- Update **upper confidence bounds** (UCBs) for all nodes:

$$\text{UCB} = \underbrace{\left| \boldsymbol{w}_i^{\top} \psi - 1 \right|}_{\text{exploitation}} + \underbrace{c \, ||\boldsymbol{w}_i||_{V_t^{-1}}}_{\text{exploration}}$$

  The exploration term is determined by our concentration result (see the paper).

- Select the top b nodes greedily
- Continue on...

Extension to sampling strategies

Theorem (Concentration of voltage within sampling strategies)

Let $\mathcal{S} \subseteq \mathcal{N}$ be a sampling of b nodes. Suppose that $\Delta_t := \Delta$ for all t, and suppose that LinDistFlow accurately represents the network model. If the assumptions hold, we have

$$\mathrm{E}\left[\max_{i\in\mathcal{S}} |v_i - 1|\right] \lesssim \frac{1}{2}\Delta \max_{i\in\mathcal{S}} \left|\left|\boldsymbol{\Lambda}^{-1}\boldsymbol{w}_i\right|\right|_2^2 \sqrt{2\log(b)}; \qquad (10)$$

moreover, for all $\epsilon > 0$

$$\Pr\left[\max_{i\in\mathcal{S}} |v_i - 1| > \epsilon\right] \leq 2b\exp\left\{\frac{-2\epsilon^2}{\Delta^2 \max_{i\in\mathcal{S}} \left|\left|\boldsymbol{\Lambda}^{-1}\boldsymbol{w}_i\right|\right|_2^2}\right\}. \qquad (11)$$

Guaranteed performance

The regret of the sampler over m periods is bounded as

$$R_m \leq \tilde{\mathcal{O}}(d\sqrt{m}), \tag{12}$$

where d is the **effective dimension** of the graph Laplacian:

$$d := \max_{i \in \mathcal{N}} \; i \quad \text{s.t.} \quad (i-1)\lambda_i \leq \frac{m}{\log(1 + m/\lambda_1)}, \tag{13}$$

where λ_1 is the smallest eigenvalue of L.

The optimal hyperparameter β depends on the effective dimension, the spectrum of the Laplacian. See our paper or[a] for more.

[a] T. Kocák, et al., "Spectral Bandits", Journal of Machine Learning Research, 21 (1), Jan. 2020.

Key take-away

Question: Why is this an improvement?

Answer: The worst case regret with standard least-squares is

$$R_m \leq \tilde{O}(n\sqrt{m}),$$

where n is the number of nodes. Our result, by **incorporating the graphical structure**[1] of the power flow equations,

$$R_m \leq \tilde{O}(d\sqrt{m}),$$

reduces the scaling factor to the *intrinsic dimension*, $d < n$, of the graph Laplacian.

(This is a huge improvement, as we will see empirically.)

[1] T. Kocák, et al., ”Spectral Bandits”, Journal of Machine Learning Research, 21 (1), Jan. 2020.

New metric: AC regret

Limitations of traditional regret metric

- The traditional regret metric uses the **linear power flow approximation** as the "ground truth" for the best voltage sampling strategy
- Robust theoretical guarantees (more on this later), but not a good empirical metric due to lack of physical realism.
- The **AC power flow (ACPF)** provides a much more realistic model of the power flow equations (non-linear).

In the power system setting we can define the (empirical) metric we term **AC regret:**

$$\text{AC regret} = \text{E}\left[\text{Clairvoyant ACPF voltage sampling strategy} - \text{Your strategy}\right]$$

Note: Involves solving a non-linear estimation problem... no guarantees

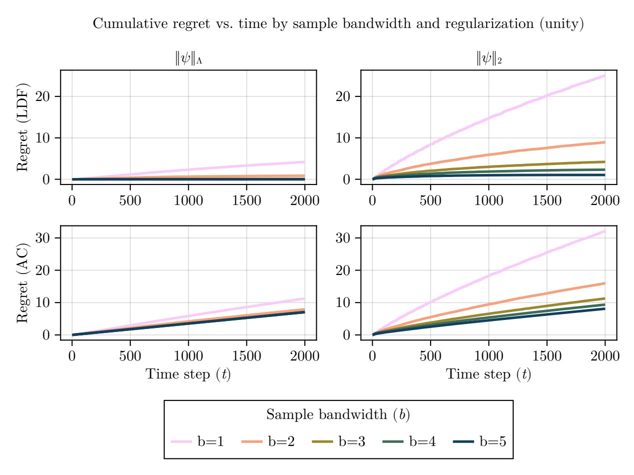


Figure 4: Fixed power factor: Regret of the bandwidth-constrained maximal voltage risk sampler vs. time with spectral (left) and ℓ_2 (right) regularization.

Additional empirical results for randomized control

We can relax the assumption on $\boldsymbol{q} = \boldsymbol{K}\boldsymbol{p}$, and let the entries of $\kappa_i := K_{ii}$ be random, e.g.,

$$\kappa_i \sim \text{Uniform}(\underline{\kappa}_i, \overline{\kappa}_i) \quad i = 1, \dots, n.$$

The following numerical results demonstrate that this works empirically, future work will generalize this.

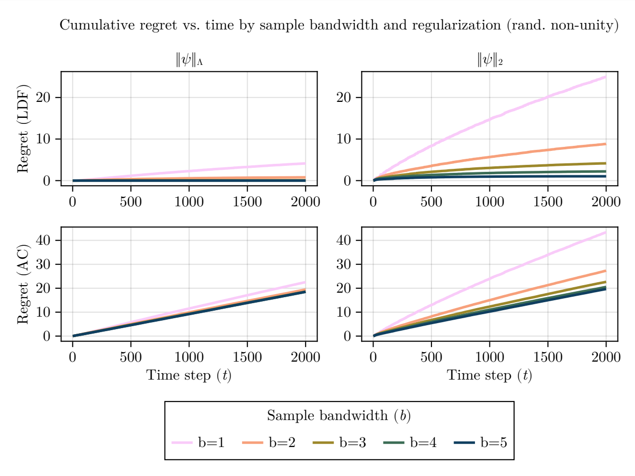


Figure 5: Non-fixed power factor: Regret of the bandwidth-constrained maximal voltage risk sampler vs. time with spectral (left) and ℓ_2 (right) regularization.

Thanks! Keep in touch: talkington@gatech.edu

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