Sampling Favorable Power Networks

Samuel Talkington

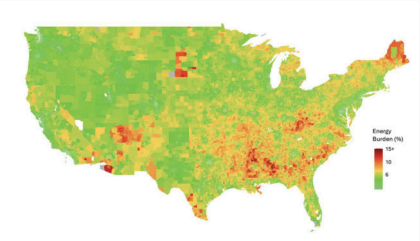
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Electricity is becoming unaffordable as demand surges



Pricing growth has unequal impacts.[a]

- Average US electricity bills **rose 4.8% per year** from 2019–2023[a]
- **1 out of 3 US households:** forgoing basic necessities (food/medicine) to pay energy bills in 2024[b]
- US utility **consumer debt totaled $21.0 billion** as of September 2024.

[a] C. Batlle et al., "US federal resource allocations are inconsistent with concentrations of energy poverty," *Sci. Adv.* **10**, 2024.

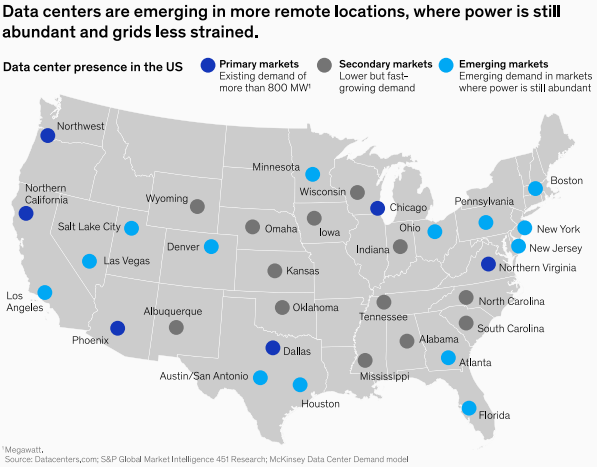
[a] S. Forrester, et al., "Retail Electricity Price and Cost Trends," Lawrence Berkeley National Laboratory, 2024.

[b] U.S. Census Phase 4.2 Cycle 09 Household Pulse Survey: August 20 – September 16, 2024.

Graph algorithms for the power grid

Electricity demand is growing, and the grid must expand.

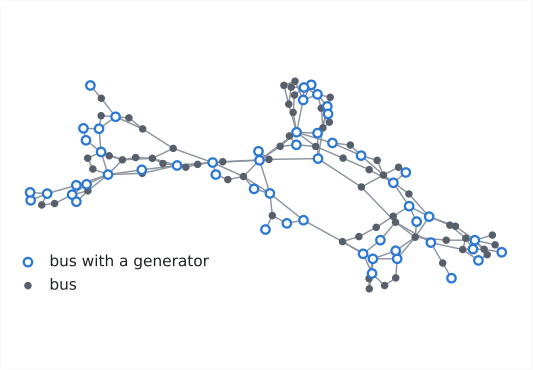
Operation and planning require graph structure.



AC power flow equations

- A grid is an undirected graph $G = (\mathcal{N}, \mathcal{E})$: n nodes, m lines
- **Voltage phasors** $\boldsymbol{u} = \boldsymbol{v}\angle\boldsymbol{\theta} \in \mathbb{C}^n$;
- complex power **injections** $\boldsymbol{s} = \boldsymbol{p} + \mathrm{j}\boldsymbol{q} \in \mathbb{C}^n$
- **Admittance matrix** $\boldsymbol{Y} = \boldsymbol{G} + \mathrm{j}\boldsymbol{B} \in \mathbb{C}^{n\times n}$: a complex-valued graph Laplacian matrix[a]

[a] up to diagonal perturbations



$$\boldsymbol{s} = \operatorname{diag}(\boldsymbol{u})\,\underline{\boldsymbol{Y}}\,\boldsymbol{u}$$

power injections, topology; nonlinear: couples every node

Linearizing the power flow equations

Linear power flow model

A first-order Taylor expansion at *nominal* conditions ($\boldsymbol{u} = \mathbf{1}$) gives

$$\begin{bmatrix} p \\ q \end{bmatrix} \approx \begin{bmatrix} G & -B \\ -B & -G \end{bmatrix} \begin{bmatrix} v - 1 \\ \theta \end{bmatrix} \iff \begin{bmatrix} v - 1 \\ \theta \end{bmatrix} \approx \begin{bmatrix} R & X \\ X & -R \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix}, \tag{1}$$

where $G \succ \mathbf{0}$, $B \prec \mathbf{0}$ are the real and imaginary components of the (reduced) admittance matrix Y, and $R, X \succ 0$ are the *resistance and reactance* matrices.

DC approximation: $G = \mathbf{0}$ and no shunts give $p \approx L\theta$, with:

$L := -B := AWA^\top$ the negative susceptance (Laplacian) matrix,

A the incidence matrix (column a_e for line e) and $W = \mathrm{diag}(w)$ the line susceptances.

DC optimal power flow (DC-OPF)

$$\min_{g,\theta} \; c(g) \qquad \text{total operational cost}$$

$$\text{s.t.} \quad L\theta = p := Cg - d \qquad \text{DC power flow; } C\text{: generator-to-node incidence matrix}$$

$$\theta_r = 0 \text{ (or } \theta \perp \mathbf{1}) \qquad \text{“slack bus”/angle reference}$$

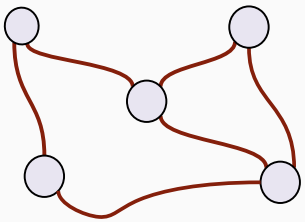
$$|w_e \, a_e^\top \theta| \le \overline{f}_e \qquad \text{limits on the flow } f_e = w_e a_e^\top \theta \text{ of every line } e$$

$$\underline{g} \le g \le \overline{g} \qquad \text{limits of every generator}$$

Here the network, and so L, is fixed. Can we choose a better network?

Stott, Jardim, Alsaç TPWRS'09.

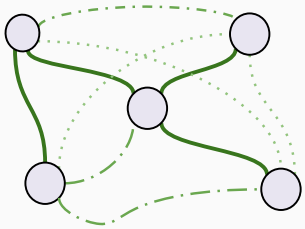
Loaded dice: The key insight



How should I connect the network?

What (single) config most reduces congestion?

Network Reconfiguration: Challenging, poorly scalable integer program



How should I connect the network?

What *family* of configs most reduces congestion?

Observation: *What if we didn't have to commit to just one configuration?*

Key idea: Optimize over a probability distribution of configurations $\rightarrow$ samples *concentrate* around "good" solutions $\rightarrow$ families of near-optimal designs.

Network reconfiguration problem

Network Reconfiguration

Find network configuration $\boldsymbol{s}$ that minimizes congestion:

$$\min_{\boldsymbol{s}\in\{0,1\}^m} \varphi(\boldsymbol{s}) \quad \text{s.t.} \quad \boldsymbol{s} \in \mathcal{S}$$

- Mixed-integer non-linear program
- **Hard in practice**, too

Example: $\varphi(\boldsymbol{s}) = \boldsymbol{p}^\mathsf{T} L_{\boldsymbol{s}}^{+} \boldsymbol{p}$ and

$$\begin{aligned}\mathcal{S} &= \{\boldsymbol{s} : \text{at most } q \text{ lines closed}\} \\ &= \{\boldsymbol{s} \in \{0,1\}^m \ : \ \|\boldsymbol{s}\|_1 \leq q\}\end{aligned}$$

Applications:

- Operational switching
- Expansion planning: New-build and reconductoring
- Siting: Data centers and storage

Our approach: Randomized switching

- **Relax:** $\boldsymbol{s} \in [0,1]^m$ (switching probabilities)
  - Works for **adding** or **closing** lines
- **Optimize:** φ is convex in $\boldsymbol{s}$, with gradient

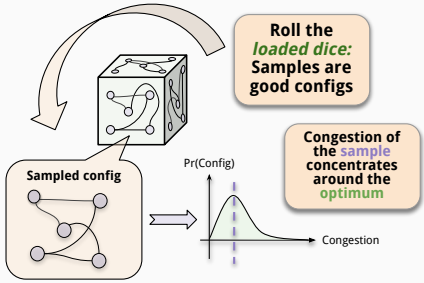
$$(\nabla\varphi(\boldsymbol{s}))_e = -w_e \left|\left\langle \boldsymbol{a}_e, \boldsymbol{L}_{\boldsymbol{s}}^{+}\boldsymbol{p}\right\rangle\right|^2 \leq 0$$

- **Sample:** $\tilde{\boldsymbol{s}} \in \{0,1\}^m$ via Bernoulli rounding

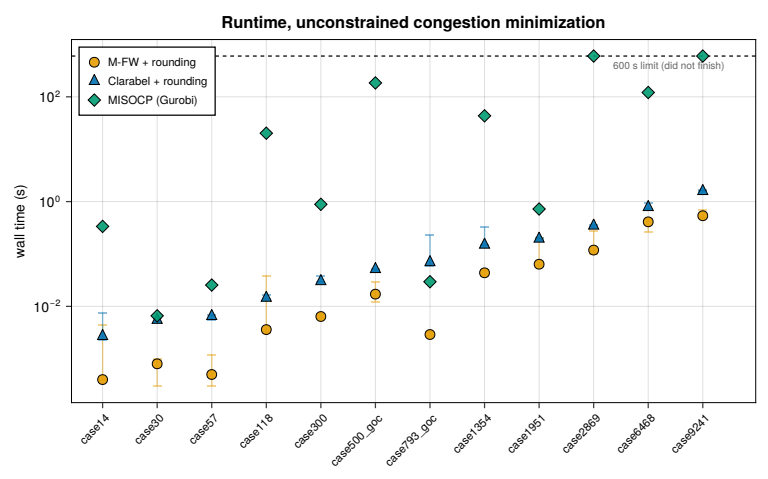
Main Result (Matrix Bernstein)

w.h.p., $\varphi(\tilde{\boldsymbol{s}}) \leq \frac{1+\alpha}{1-\epsilon}\varphi(\boldsymbol{s}_\star)$,
where $\boldsymbol{s}_\star \leftarrow$ optimal probabilities

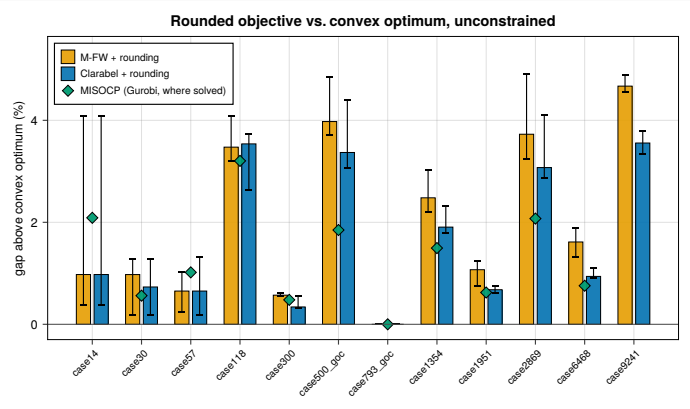
Families of near-optimal solutions.



Randomized switching is the fastest method on every network



Median rounded topology lands within 5% of the convex optimum



- Median end-to-end gap (residual fractional suboptimality **plus** rounding loss) is within 5% of the convex optimum on all 12 cases

Expansion planning generalizes switching with an investment cost

Expansion Planning

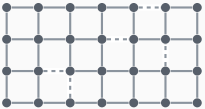
$$\min_{\boldsymbol{s}\in[0,1]^K,\ ||\boldsymbol{s}||_1\le q} \kappa \mathbf{1}^\mathsf{T}\boldsymbol{s} + \varphi(\boldsymbol{s}), \qquad \varphi(\boldsymbol{s}) = \boldsymbol{p}^\mathsf{T} L_{\boldsymbol{s}}^{+} \boldsymbol{p}, \qquad L_{\boldsymbol{s}} = L_0 + \sum_{k=1}^{K} s_k w_k \boldsymbol{a}_k \boldsymbol{a}_k^\mathsf{T}$$

- L_0: the Laplacian of the existing network. s_k: the built fraction of candidate line k.
- κ: build cost per line. With $\kappa = 0$, building is free.

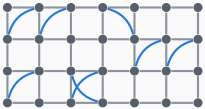
Garver TPAS'70; Ghosh, Boyd, Saberi SIREV'08; **Talkington**, Ostrovskii, Molzahn arXiv'25.

Reconfiguration opens lines; expansion planning adds them

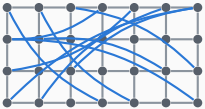
open lines



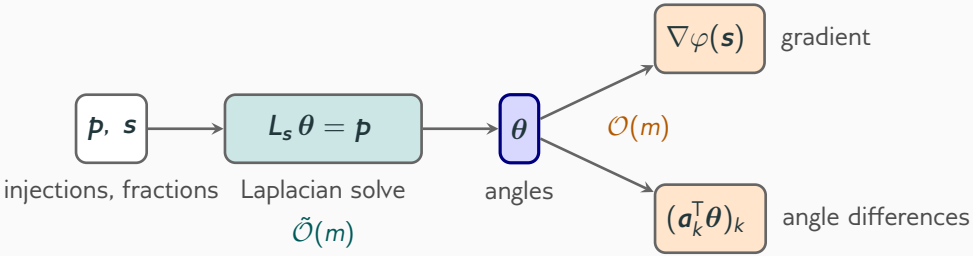
new local lines



new random lines



Each iteration uses one Laplacian solve



Frank-Wolfe for expansion planning

$\boldsymbol{s} \leftarrow \mathbf{0}$

for $t = 0, 1, \ldots$ **do**

- $\boldsymbol{\theta} \leftarrow \boldsymbol{L}_{\boldsymbol{s}}^{+} \boldsymbol{p}$ ▷ one Laplacian solve
- $h_k \leftarrow \kappa - w_k (\boldsymbol{a}_k^{\mathsf{T}} \boldsymbol{\theta})^2$ for each candidate k ▷ the gradient
- $\boldsymbol{z} \leftarrow$ indicator of the q most negative entries of $\boldsymbol{h}$ (negative only) ▷ linear minimization
- $\boldsymbol{s} \leftarrow \boldsymbol{s} + \frac{2}{t+2}(\boldsymbol{z} - \boldsymbol{s})$

Then round $\tilde{s}_k \sim \mathrm{Ber}(s_k)$ independently.

Frank, Wolfe NRLQ'56.

Iteratively reweighted least squares (IRLS): alternate flows and weights

$\varphi(\boldsymbol{s}) = \min_{\boldsymbol{f} \text{ routes } \boldsymbol{p}} \sum_e f_e^2/w_e + \sum_k f_k^2/(s_k w_k)$ over existing lines e and candidates k, minimized by the flow $f_k = s_k w_k \, \boldsymbol{a}_k^\top \boldsymbol{\theta}$, with $\boldsymbol{\theta} = \boldsymbol{L}_{\boldsymbol{s}}^{+} \boldsymbol{p}$.

$\boldsymbol{s} \leftarrow (q/K)\mathbf{1}$ ▷ the budget spread evenly

for $t = 0, 1, \ldots$ **do**

- $\boldsymbol{\theta} \leftarrow \boldsymbol{L}_{\boldsymbol{s}}^{+} \boldsymbol{p}$ ▷ best $\boldsymbol{f}$ for fixed $\boldsymbol{s}$: one Laplacian solve
- $f_k \leftarrow s_k w_k \, \boldsymbol{a}_k^\top \boldsymbol{\theta}$ for each candidate k
- $s_k \leftarrow \min\{1, \ \max\{\delta, \ |f_k|/\sqrt{w_k(\kappa + \lambda)}\}\}$ ▷ best $\boldsymbol{s}$ for fixed $\boldsymbol{f}$

λ: the smallest $\lambda \geq 0$ for which the new $\boldsymbol{s}$ satisfies $||\boldsymbol{s}||_1 \leq q$. $\delta = 10^{-6}$: a floor on s_k.

Suggested by R. Kyng. Straszak, Vishnoi ITCS'17; Adil, Peng, Sachdeva NeurIPS'19; Ene, Vladu ICML'19; Ene, Nguyen, Vladu ICLR'26; Beck SIOPT'15.

Laplacian solvers in nearly linear time

Fast DC power flow

For $\boldsymbol{L}$ with $\mathrm{nnz}(\boldsymbol{L}) = \mathcal{O}(m)$, there is an algorithm Solve$(\boldsymbol{L}, \boldsymbol{p}, \epsilon)$ returns $\hat{\boldsymbol{\theta}}$ with

$$\left|\left|\hat{\boldsymbol{\theta}} - \boldsymbol{L}^{+}\boldsymbol{p}\right|\right|_{\boldsymbol{L}} \leq \epsilon \left|\left|\boldsymbol{L}^{+}\boldsymbol{p}\right|\right|_{\boldsymbol{L}}$$

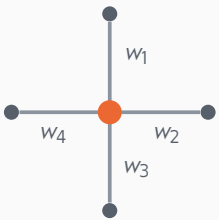
w.p. $\geq 1 - \mathcal{O}(\delta)$, in time $\mathcal{O}\left(m \log^2(1/\delta) \log n \log(1/\epsilon)\right) = \tilde{\mathcal{O}}(m)$.

Idea: Build a random Cholesky factor $\boldsymbol{C}$ such that, w.h.p. $\frac{1}{2}\boldsymbol{L} \preceq \boldsymbol{C}\boldsymbol{C}^{\star} \preceq \frac{3}{2}\boldsymbol{L}$.

Kyng, Sachdeva FOCS'16.

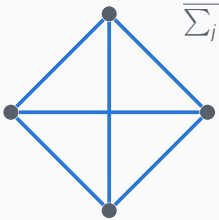
Elimination is the star-mesh transform

eliminate a node



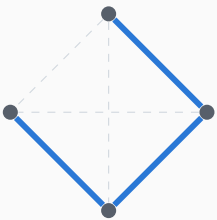
a Schur complement

a clique appears



Cholesky stores every clique

or a sample of it



approximate Cholesky: a few edges, unbiased

Rosen J. IEE'24; Kron '39; Kyng, Sachdeva FOCS'16.

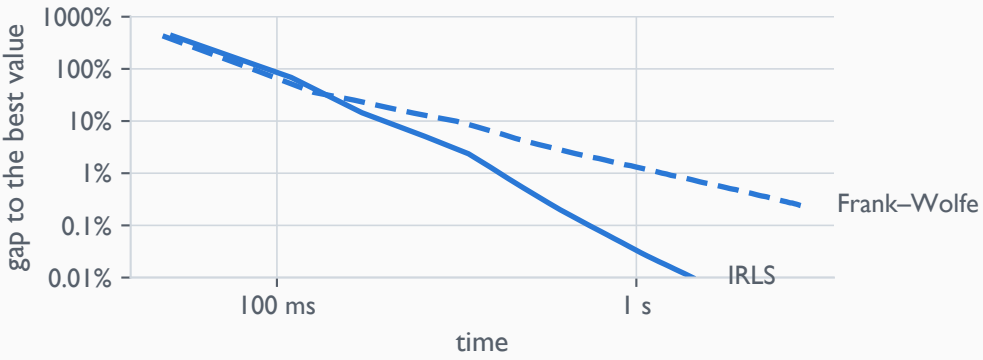
Expansion planning with 10% random candidates

	time per iteration (ms)			
nodes	Cholesky of L_s	apxchol of L_s	PCG, Cholesky of L_0	speedup
30k	31	**16**	19	1.2×
78k	390	**35**	66	1.9×
120k	1381	**45**	154	3.4×

Speedup: apxchol against the next fastest.

apxchol: Kulkov, Baumann, Kyng, software '26.

IRLS gets within 1% of the best value 2.8× sooner than Frank–Wolfe



120k nodes, 10% random candidates.

Summary

- Optimize a distribution over networks, then sample: near-optimal w.h.p. (matrix Bernstein).
- Switching: fastest on every test network; median within 5% of the convex optimum.
- Planning (10% random candidates): apxchol 1.2–3.4$\times$ faster per iteration.
- IRLS: a 1% gap 2.8$\times$ sooner than Frank–Wolfe (120k nodes).

Thank you! Questions?

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