Randomness as a Resource for Electric Power Systems

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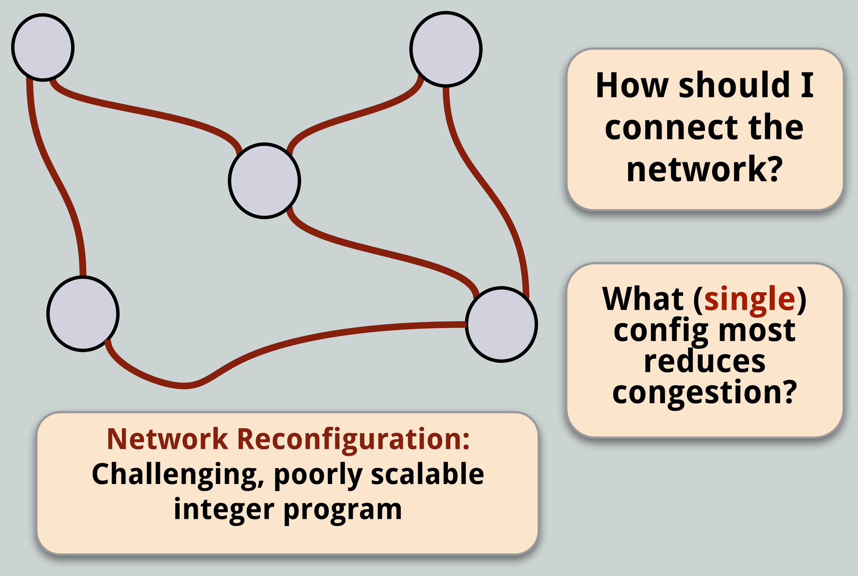
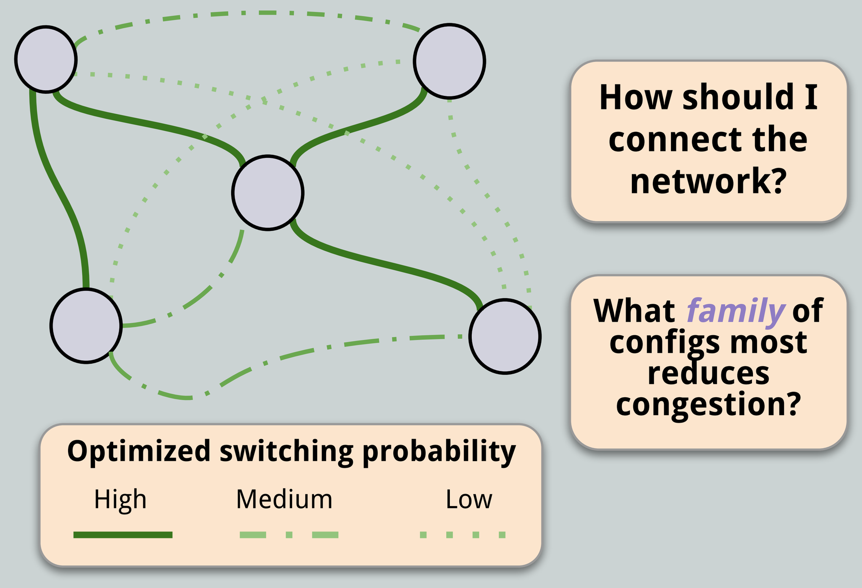
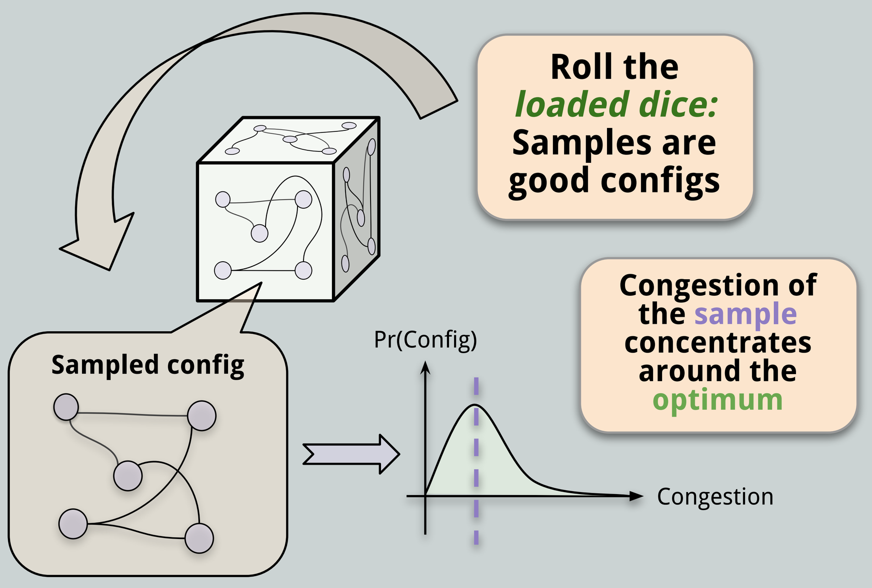
Contribution: Randomized Switching Framework

Efficient algorithm for the electrical network reconfiguration problem achieving **100–1000× speedup** over commercial MISOCP solvers.







Key idea: Optimize over *probability distribution* of network configurations.

Mathematical Framework

Switchable network

- Undirected graph $G = (N, E, \mathbf{w})$; $n = |N|$ nodes and $m = |E|$ *potential edges*.
- Nodal demands $\mathbf{d} \perp \mathbf{1}_n$; edge *switching decisions* $\mathbf{s} \in \{0,1\}^m$.
- Laplacian matrix functional: $\mathbf{L}_\mathbf{s} := \sum_{e \in E} s_e w_e \mathbf{a}_e \mathbf{a}_e^\top$; where $\mathbf{a}_e = \chi_i - \chi_j$ is the incidence vector of edge $e = (i,j) \in E$.
- *Connected template:* $T \subseteq E$ such that $s_e = 1$ for all $e \in T$, and $\lambda_2(\mathbf{L}_T) > 0$.

Electrical network reconfiguration problem

The electrical network reconfiguration problem is a mixed-integer non-linear program (MINLP) of the form

$$\min_{\mathbf{s} \in \{0,1\}^m} \varphi(\mathbf{s}) = \mathbf{d}^\top \mathbf{L}_\mathbf{s}^\dagger \mathbf{d} \qquad \text{s. t.} \qquad \|\mathbf{s}\|_1 \le q, \quad s_e = 1 \ \ \forall e \in T.$$

Applications: Managing electrical congestion from data centers; Expanding electrical transmission capacity.

Congestion

$$\varphi(\mathbf{s}) = \mathbf{d}^\top \mathbf{L}_\mathbf{s}^\dagger \mathbf{d} = \sum_{(i,j) \in E} w_{ij}(x_i - x_j)^2 = \sum_{e \in E} w_e^{-1/2} f_e^2$$

Congestion $\varphi(\cdot)$ is a **homogeneous** function of degree -1, analogous to the total effective resistance. It is also generalized self-concordant.

Optimization Procedure

Algorithm 1: Monotonic Frank-Wolfe with randomized rounding for efficient network reconfiguration.

Input: $\mathbf{s}_0$, $\mathbf{d} \perp \mathbf{1}$, E, $\mathbf{w}$, q, T, ϵ, δ.
Output: Random integral switching strategy $\tilde{\mathbf{s}}$, induced voltage phasors $\tilde{\mathbf{s}} \mapsto \tilde{\mathbf{x}}$.

function RANDRECONFIG$(T, E, q, \mathbf{w}, \mathbf{d}, \epsilon, \delta)$
  $\mathbf{L}_{\mathbf{s}_0} \leftarrow \sum_{ij \in \mathrm{supp}(\mathbf{s}_0)} \mathbf{E}_{ij} w_{ij}$ // $O(m)$
  for $t = 0, 1, \ldots,$ **to** $T - 1$ **do**
    $\eta_t \leftarrow \frac{2}{t+2}$ // Set step size
    // -- compute the congestion gradient --
    $\nabla\varphi(\mathbf{s}_t), \Delta(\mathbf{s}_t) \leftarrow$ APPROXDIFF$(\mathbf{s}_t, \mathbf{d}, \mathbf{w}, E, \epsilon, \delta)$
    // -- find a vertex --
    $\mathbf{v}_t \leftarrow \arg\min_{\mathbf{v} \in \{0,1\}^m, \|\mathbf{v}\|_1 \le q} \langle \nabla\varphi(\mathbf{s}_t), \mathbf{v} \rangle$
    // -- update convex combination --
    $\mathbf{s}_{t+1} \leftarrow (1 - \eta_t)\mathbf{s}_t + \eta_t \mathbf{v}_t$
  end
  // -- round the fractional solution --
  $\tilde{\mathbf{s}} \leftarrow$ ROUND$(\mathbf{s}_T)$ // $\tilde{O}(m)$
  // -- solve for the voltages --
  $\tilde{\mathbf{x}} \leftarrow$ Solve $(\mathbf{L}_{\tilde{\mathbf{s}}}, \mathbf{d}, \delta, \epsilon)$
  return $\tilde{\mathbf{s}}, \tilde{\mathbf{x}}$
end

This algorithm runs in $\tilde{O}(mq/\alpha)$ time for an α-approximation of the optimal probabilities, bicriterion emerges for rounding (see above).

References

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[2] D. A. Spielman, N. Srivastava. "Graph Sparsification by Effective Resistances." *SIAM J. Comput.* 40(6), 2011.
[3] A. Ghosh, S. Boyd and A. Saberi. "Minimizing Effective Resistance of a Graph." *SIAM Review*, 50(1):37-66, 2008.
[4] A. Carderera, M. Besançon, and S. Pokutta. "Scalable Frank–Wolfe on Generalized Self-Concordant Functions via Simple Steps." *SIAM J. Opt.*, 34(3):2231-2258, 2024.

Concentration

Randomized Rounding

1. **Relax:** $\mathbf{s} \in [0,1]^m$ (switching *probabilities*)
2. **Optimize:** Frank-Wolfe with gradient (leverage scores!)

$$(\nabla\varphi(\mathbf{s}))_e = -w_e \left| \langle \mathbf{a}_e, \mathbf{L}_\mathbf{s}^\dagger \mathbf{d} \rangle \right|^2 \le 0$$

3. **Sample:** $\tilde{\mathbf{s}} \in \{0,1\}^m$ via Bernoulli rounding

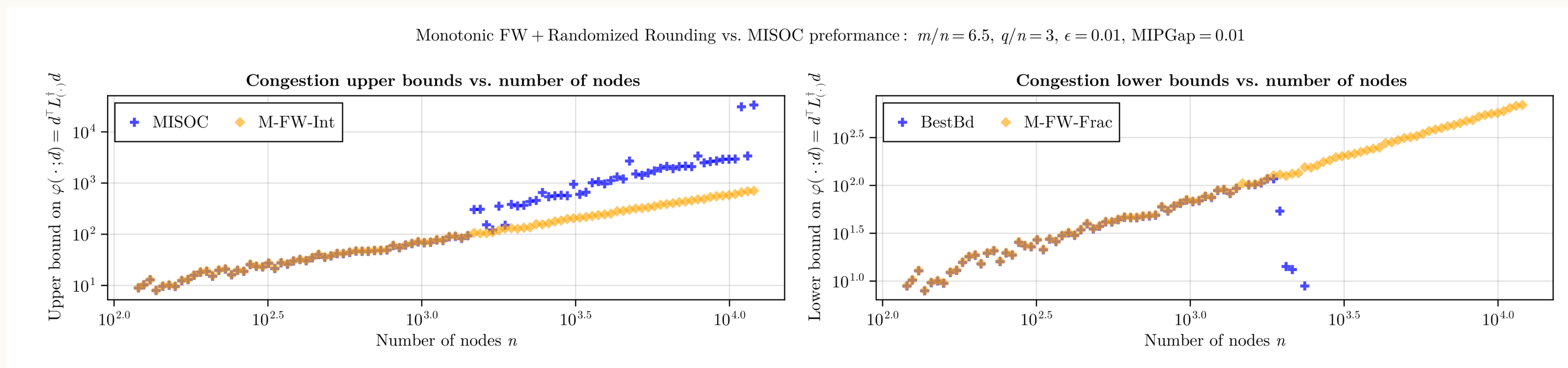
Theorem 1. *Assume that the fixed backbone $T \subseteq E$ satisfies $\|\mathbf{d}\|^2_{\mathbf{L}_T^\dagger} \le 1$. For any $\delta \in (0,1)$, Algorithm* RANDRECONFIG *returns a random integral solution $\tilde{\mathbf{s}} \in \{0,1\}^m$ that satisfies, with probability at least $1 - \delta$,*

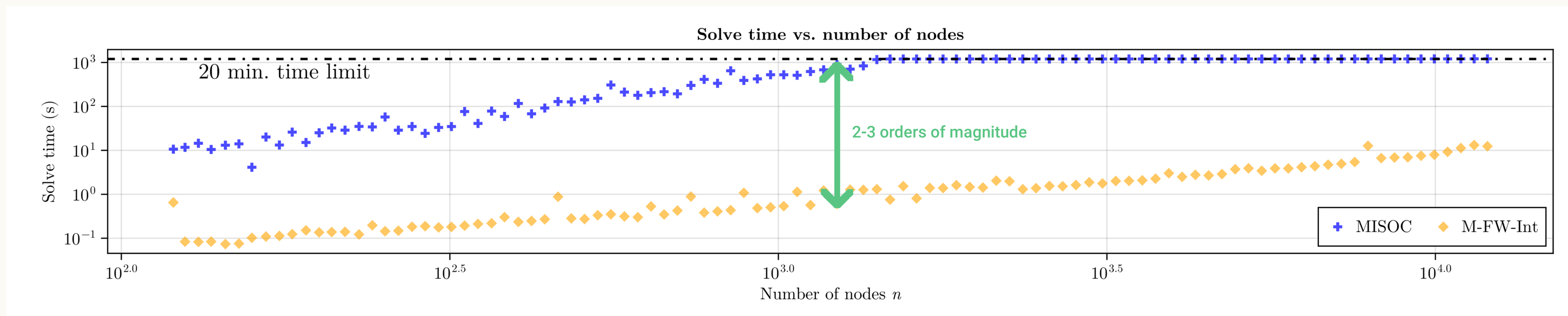
$$\varphi(\tilde{\mathbf{s}}) \le \frac{1}{1-\epsilon}\varphi(\mathbf{s}_t) \le \frac{1+\alpha}{1-\epsilon}\varphi(\mathbf{s}_\star), \quad \text{with} \quad \epsilon \lesssim \sqrt{\log(n/\delta)}$$

and $\mathbb{E}\|\tilde{\mathbf{s}}\| = q$, in total running time $\tilde{O}(mq/\alpha)$, where $\tilde{O}(\cdot)$ suppresses polylogarithmic factors in n and the Laplacian solver tolerance.

First-of-its-kind guarantee for network reconfiguration!

Numerical Results





Key Findings

- **100–1000× speedup** over Gurobi MISOCP
- Scales to n = 12,000+ nodes (Gurobi times out)
- Solution quality comparable to MISOCP optimum

Resources & Contact

Interactive Demo

arXiv Preprint

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