

Admittance Matrix Concentration Inequalities for Understanding Uncertain Power Networks

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PowerUp 2026

Boulder, Colorado

September 2026

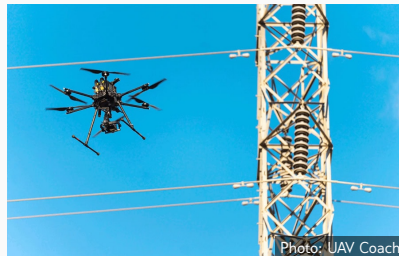
Power grids themselves are often not known for certain

Storms take lines out without warning.



Hurricane Ike

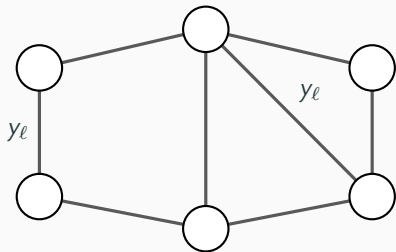
New forms of risk.



Drones+transmission lines=?

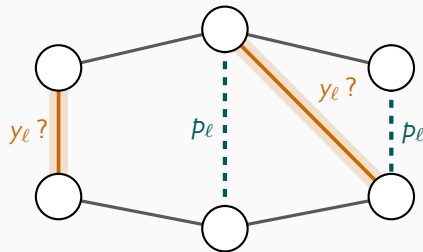
Also measurement error in line data, power flow control devices, and deliberate attacks.

Traditional model



Known lines, known admittances: Y fixed.

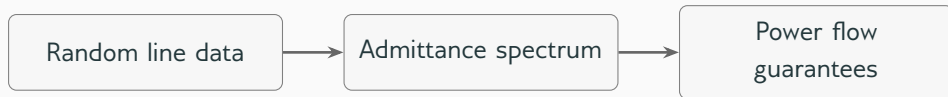
This work



Uncertain lines and admittances: Y random.

The admittance matrix becomes random.

Perspective	Random object	Typical question
Probabilistic power flow	Power injections	How do injections affect the state?
Cascading failures	Sequence of line failures on a graph	How do failures spread?
This work	Complex admittance matrices	How does network uncertainty affect power flow?



Probabilistic power flow: Borkowska, 1974. Cascading failures: Watts, PNAS 2002; Hines, Cotilla-Sanchez, and Blumsack, Chaos 2010. Random Laplacians: Oliveira, arXiv:0911.0600; Le, Levina, and Vershynin, 2017.

Scalar concentration

$$\Pr(|Z - \mathbb{E} Z| \geq t) \leq \delta(t).$$



Chance of straying farther than t

Matrix concentration

$$\Pr(\|Y - \mathbb{E} Y\|_2 \geq t) \leq \delta(t).$$

The operator norm is the largest response to any unit input:

$$\|M\|_2 = \sup_{\|x\|_2=1} \|Mx\|_2.$$

A tail bound also bounds the **average**, since for $W \geq 0$, $\mathbb{E} W = \int_0^\infty \Pr(W > t) dt$.

Independent, symmetric $X_1, \dots, X_k \in \mathbb{R}^{d \times d}$ with $\mathbb{E} X_\ell = 0$ and $\|X_\ell\|_2 \leq R$.

$$X = \sum_{\ell=1}^k X_\ell, \quad \nu = \left\| \sum_{\ell=1}^k \mathbb{E}[X_\ell^2] \right\|_2.$$

Matrix Bernstein

$$\Pr(\|X\|_2 \geq t) \leq 2d \exp\left(-\frac{t^2/2}{\nu + Rt/3}\right), \quad t > 0.$$

$$\mathbb{E} \|X\|_2 \leq \sqrt{2\nu \log(2d)} + \frac{R}{3} \log(2d).$$

R : largest single term

ν : total variance

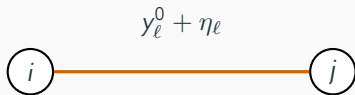
d : matrix size

J. A. Tropp, *An Introduction to Matrix Concentration Inequalities*, Found. Trends Mach. Learn., 2015.

Fixed lines, random admittances

$$w_\ell = y_\ell^0 + \eta_\ell, \quad \mathbb{E} \eta_\ell = 0, \quad |\eta_\ell| \leq r.$$

- Independent line perturbations η_ℓ .
- Any bounded distribution.
- Bounded and zero mean, hence sub-Gaussian.



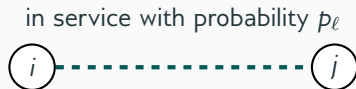
$$\text{where } Y = \sum_\ell w_\ell E_\ell, \quad \tilde{Y} := Y - \mathbb{E} Y.$$

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Theorems 2-3**.

Fixed admittances, random lines

$$w_\ell = y_\ell \xi_\ell, \quad \xi_\ell \sim \text{Bernoulli}(p_\ell).$$

- Independent on/off statuses ξ_ℓ .
- Each line has its own probability p_ℓ , so **inhomogeneous Erdos-Rényi** graph.



Line $\ell = (i, j)$ contributes the elementary Laplacian

$$E_\ell = (e_i - e_j)(e_i - e_j)^T, \quad E_\ell|_{\{i,j\}} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

With $w_\ell = g_\ell + \mathrm{j}b_\ell$ and $Y = G + \mathrm{j}B$, write Y as one real symmetric matrix, the lift

$$\mathcal{H}(Y) := \begin{bmatrix} G & B \\ B & -G \end{bmatrix} = \sum_{\ell \in E} \begin{bmatrix} g_\ell & b_\ell \\ b_\ell & -g_\ell \end{bmatrix} \otimes E_\ell.$$

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Section II**. Undirected model with no shunts or taps.

n buses, at most Δ lines at any bus. Independent zero mean line errors η_ℓ with $|\eta_\ell| \leq 1$.

Theorem 2: bounded admittance errors on fixed lines

$$\mathbb{E} \left\| \tilde{Y} \right\|_2 \leq \sqrt{8\Delta \log(4n)} + \frac{2}{3} \log(4n).$$

- Any independent, bounded line errors; no distribution shape assumed.
- Only logarithmic growth in the number of buses n .
- Errors up to r : multiply the bound by r .

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Theorem 2**. Applied to the centered admittance perturbation.

Fixed $|y_\ell| \leq 1$; independent $\xi_\ell \sim \text{Bernoulli}(p_\ell)$.

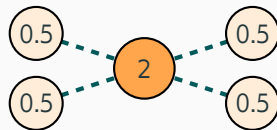
$$\underbrace{c_\ell = 2p_\ell(1-p_\ell)|y_\ell|^2}_{\text{line}} \quad \underbrace{d_i(c) = \sum_{\ell \ni i} c_\ell}_{\text{bus}} \quad \underbrace{\Delta_c = \max_i d_i(c)}_{\text{largest bus}} \quad \underbrace{\bar{D} = \frac{\sum_i d_i(c)}{\Delta_c} \leq n}_{\text{normalized total}}$$

Spread along a path



$$\Delta_c = 1, \quad \bar{D} = 4$$

Concentrated at a hub



$$\Delta_c = 2, \quad \bar{D} = 2$$

Same four lines with $c_\ell = 1/2$: the hub doubles Δ_c and halves $\bar{D}$.

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Definition 3**. Contingency factors and nodal criticality.

$$\tilde{Y} = \sum_{\ell \in E} (\xi_\ell - p_\ell) y_\ell E_\ell; \text{ independent on/off statuses, } |y_\ell| \leq 1, \Delta_c > 0.$$

Theorem 3: independent random line outages

For $t \geq \sqrt{2\Delta_c} + 2/3$,

$$\Pr(\|\tilde{Y}\|_2 \geq t) \leq 8\bar{D} \exp\left(-\frac{t^2}{4(\Delta_c + t/3)}\right).$$

$$\mathbb{E} \|\tilde{Y}\|_2 \leq C \left(\sqrt{2\Delta_c \log(1 + 2\bar{D})} + 2 \log(1 + 2\bar{D}) \right).$$

with a universal constant $C > 0$.

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Theorem 3**. Intrinsic dimension Bernstein.

Lifted admittance $\mathcal{H}(Y)$

$$\mathcal{H}(Y) = \begin{bmatrix} G & B \\ B & -G \end{bmatrix}.$$

Flat start Jacobian F

$$F = \begin{bmatrix} G & -B \\ -B & -G \end{bmatrix}.$$

$Q = \text{diag}(I_n, -I_n)$ flips the sign of the block and preserves its norm $F = Q\mathcal{H}(Y)Q$.

$$\|F\|_2 = \|Y\|_2, \quad \|F - EF\|_2 = \|Y - EY\|_2.$$

$$\underbrace{\begin{bmatrix} p \\ q \end{bmatrix}}_{\text{Linear Coupled Power Flow (LCPF)}} \approx F \underbrace{\begin{bmatrix} v-1 \\ \theta \end{bmatrix}}_{\text{LinDistFlow sensitivity form}}$$

$$\underbrace{v-1 \approx Rp + Xq}_{\text{LinDistFlow sensitivity form}}.$$

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Section II and Appendix A**.

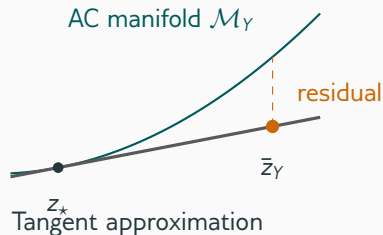
Complex voltages $u \in \mathbb{C}^n$ in rectangular coordinates, injections $s \in \mathbb{C}^n$

$$\Phi_Y(u) = \text{diag}(u)\overline{Y}u, \quad \mathcal{M}_Y = \{(u, \Phi_Y(u)) : u \in \mathbb{C}^n\} \subset \mathbb{R}^{4n}.$$

From $z_\star = (u_\star, \Phi_Y(u_\star))$, a step h along the tangent is

$$\bar{z}_Y = z_\star + (h, D\Phi_Y(u_\star)[h]).$$

Bound the average distance to the AC manifold over random networks.



S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, Section IV.

Fixed lines, with the independent zero mean line errors of Theorem 2 ($|\eta_\ell| \leq 1$).

Proposition 1: expected distance to the AC manifold

$$\begin{aligned} \mathbb{E} \operatorname{dist}(\bar{z}_Y, \mathcal{M}_Y) &\leq 3 \|h\|_\infty \|h\|_2 \mathbb{E} \|Y\|_2 \\ &\leq 3 \|h\|_2^2 \left(\sqrt{8\Delta \log(4n)} + \frac{2}{3} \log(4n) \right). \end{aligned}$$

For a physical network $Y = Y_0 + \tilde{Y}$, keep the nominal term $\mathbb{E} \|Y\|_2 \leq \|Y_0\|_2 + \mathbb{E} \|\tilde{Y}\|_2$.

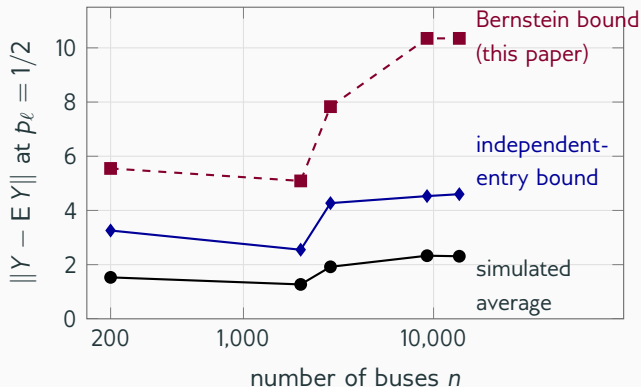
S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Proposition 1**.

Lossless network $Y = -jL_\beta$, $L_\beta = \sum_\ell \beta_\ell \xi_\ell E_\ell$, $0 < \beta_\ell \leq 1$. Independent $\xi_\ell \sim \text{Bern}(p_\ell)$.

Proposition 2: AC manifold distance under random line outages

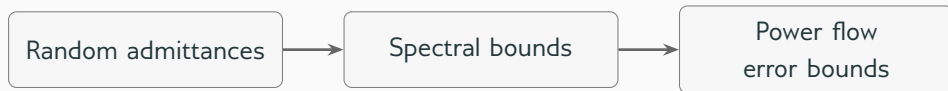
$$\mathbb{E} \text{ dist}(\bar{z}_Y, \mathcal{M}_Y) \leq 3 \|h\|_\infty \|h\|_2 \left[C \left(\sqrt{2\Delta_c \log(1 + 2\bar{D})} + 2 \log(1 + 2\bar{D}) \right) \right].$$

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Proposition 2**.



- Five transmission test cases, 200 to 13,659 buses.
- Each line in service independently with $p_\ell = 1/2$; $\max_\ell |y_\ell| = 1$.
- Simulated average between 1.3 and 2.3; the bounds sit $2\times$ to $4.5\times$ above it.

Independent entry bound from A. S. Bandeira and R. van Handel, *Sharp nonasymptotic bounds on the norm of random matrices with independent entries*, Ann. Probab., 2016.



- **Analyze existing algorithms:** average error across a family of networks.
- **Design randomized algorithms:** sampling tailored to a problem, with guarantees.
- **Study grid decisions:** probabilistic operations, reliability, and planning.

These directions are becoming new results.



Randomized switching

Design switching probabilities with controlled reconfiguration cost.

*Efficient Network
Reconfiguration by Randomized
Switching*



Quantum PF limits

Grid structure makes the equations ill-conditioned, which limits quantum speedups.

*Proving the Limits of Quantum
Power Flow*

Thank you! Questions

Backup slides

For a centered line error η_ℓ , $X_\ell = \begin{bmatrix} \operatorname{Re}\{\eta_\ell\} & \operatorname{Im}\{\eta_\ell\} \\ \operatorname{Im}\{\eta_\ell\} & -\operatorname{Re}\{\eta_\ell\} \end{bmatrix} \otimes E_\ell$.

$$E_\ell^2 = 2E_\ell, \quad X_\ell^2 = 2|\eta_\ell|^2 I_2 \otimes E_\ell, \quad \|X_\ell\|_2 = 2|\eta_\ell|.$$

Bounded weights: $|\eta_\ell| \leq 1$.

Switches: $\eta_\ell = (\xi_\ell - p_\ell)y_\ell$.

$$\nu \leq 2 \left\| \sum_\ell E_\ell \right\|_2 \leq 4\Delta.$$

$$V = \sum_\ell c_\ell E_\ell, \quad \nu = \|V\|_2 \leq 2\Delta_c.$$

Use $d = 2n$, $R = 2$ in Theorem 1.

The intrinsic dimension obeys $2 \frac{\operatorname{tr} V}{\|V\|_2} \leq 2\bar{D}$.

The variance is a positive weighted Laplacian even when the summands are indefinite.

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Appendices B–C**. Back to Slides 8 and 10.

In rectangular coordinates the exact tangent residual is

$$r_Y(h) = \Phi_Y(u_\star + h) - \Phi_Y(u_\star) - D\Phi_Y(u_\star)[h] = \text{diag}(h)\overline{Y}h.$$

Write $Y_0 = \mathbb{E} Y$. At a common fixed step h ,

$$\mathbb{E} \|r_Y(h) - r_{Y_0}(h)\|_2 \leq \|h\|_\infty \|h\|_2 \mathbb{E} \|Y - Y_0\|_2,$$

$$\mathbb{E} \|r_Y(h)\|_2 \leq \|h\|_\infty \|h\|_2 (\|Y_0\|_2 + \mathbb{E} \|Y - Y_0\|_2).$$

The same voltage AC point gives $\text{dist}(\bar{z}_Y, \mathcal{M}_Y) \leq \|r_Y(h)\|_2$.

If every line is deterministic, the fluctuation term vanishes; nominal AC curvature remains.

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Propositions 1–2**. Clarification. Back to Slides 13–14.

Delete the slack bus column from the branch-to-bus incidence matrix of a tree: A_r is square and invertible. With line impedances $z_\ell = r_\ell + jx_\ell$,

$$Y_r = A_r^T \text{diag}(1/z_\ell) A_r, \quad Y_r^{-1} = A_r^{-1} \text{diag}(z_\ell) A_r^{-T} = R + jX.$$

$$\begin{bmatrix} v - 1 \\ \theta \end{bmatrix} \approx \begin{bmatrix} R & X \\ X & -R \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix}.$$

The first block row is the voltage magnitude LinDistFlow relation.

For squared magnitudes, $v^2 - 1 \approx 2(Rp + Xq)$.

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Appendix A**. Unit slack voltage, no shunts or taps. Back to Slide 11.