Sample Complexity of Distribution Network Sensing

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Project Background

- The proliferation of smart meters is a pivotal opportunity to increase electric distribution network visibility, reaching 107 million units in the US in 2021 [1].
- Bandwidth limitations pose a significant challenge for real-time monitoring purposes, especially when limited bandwidth is shared by many smart meters.
- New smart meters can be dynamically *queried*, and their *precision* can be controlled remotely via two-way communication [2]. Variable subsets of the meters can report data at high frequencies.
- By strategically tuning these parameters, we can use these measurements to efficiently and accurately **learn an unknown grid's topology** [3] and perform **state estimation** [4].

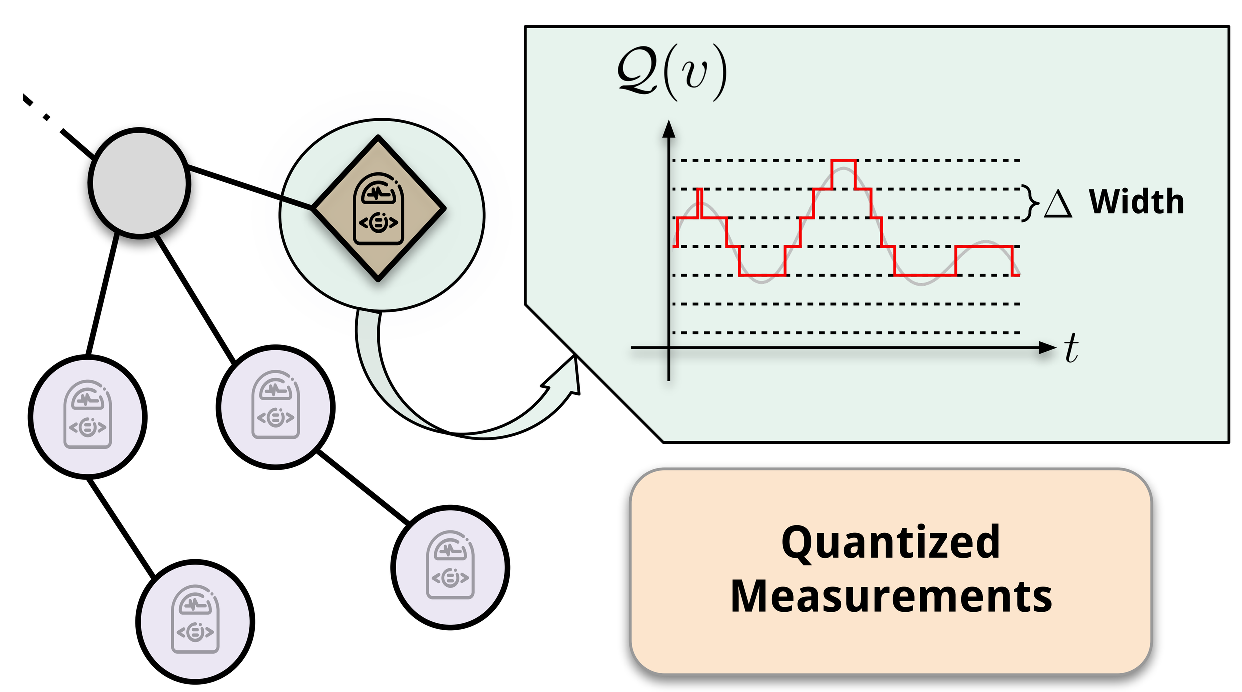

Figure 1. Quantization techniques assign continuous measurements to discrete buckets to improve computation and communication efficiency [5]. Each bucket's size is defined by the quantization width $\Delta > 0$.

Project Objectives and Contributions

Objectives:

- **Quantify the error** of a broad class of state estimation problems in power systems.
- **Quantify the impact of variable precision** in smart meter measurements on estimation and control tasks.
- **Dynamically identify smart meters to query** with objectives such as revealing violations of voltage magnitude limits and improving state estimation accuracy.

Contributions:

- **An efficient topology learning algorithm** that is robust to the probability distributions of the smart meter measurements.
- **Probabilistic bounds on the accuracy of the estimated topology** by the algorithm. This is achieved by developing concentration inequalities for the smart meter measurements under quantization effects.
- **Measurement sampling prescriptions** to ensure a desired error. This prescription depends on the quantization rates chosen for the meters.

Research Approach

Problem description: We consider an estimation problem with *quantized* linear measurements:

$$\boldsymbol{y} = \mathcal{Q}(\boldsymbol{H}\boldsymbol{x}), \tag{1}$$

where $\boldsymbol{H} \in \mathbb{R}^{m \times n}$ is a sensing matrix, $\boldsymbol{x} \in \mathbb{R}^n$ is a grid signal we wish to estimate, and $\mathcal{Q}(\cdot)$ is a *non-linear quantization function*.

Example measurement model: Assume that each y_i is a *uniformly dithered* quantized measurement with quantization width $\Delta > 0$; this means that each y_i takes the form

$$y_i = \mathcal{Q}(\langle \boldsymbol{h}_i, \boldsymbol{x}\rangle) = \Delta\left(\left\lfloor \frac{\langle \boldsymbol{h}_i, \boldsymbol{x}\rangle + \tau_i}{\Delta} \right\rfloor + \frac{1}{2}\right) \quad i = 1, \dots, m, \tag{2}$$

where $\tau_i \sim \mathsf{Uniform}\left(-\frac{\Delta}{2}, \frac{\Delta}{2}\right)$. Then, we can reformulate $\boldsymbol{y}$ as a *linear* measurement model:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{z}, \tag{3}$$

where $\boldsymbol{z} \sim \mathsf{subG}^n(\Delta)$ is a *sub-Gaussian* noise vector [6, 7].

Convex Estimation Problem: We consider a generalized constrained estimation problem:

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x} \in \mathcal{K}} \frac{1}{2m} \sum_{i=1}^{m} (\langle \boldsymbol{h}_i, \boldsymbol{x}\rangle - y_i)^2, \tag{4}$$

where $\mathcal{K} \subset \mathbb{R}^n$ is a convex set of interest that encodes *known structure* of the state.

Example: Topology Recovery

Task: Learn a n-node distribution network topology $\boldsymbol{x}_0$ from m smart meter measurements.

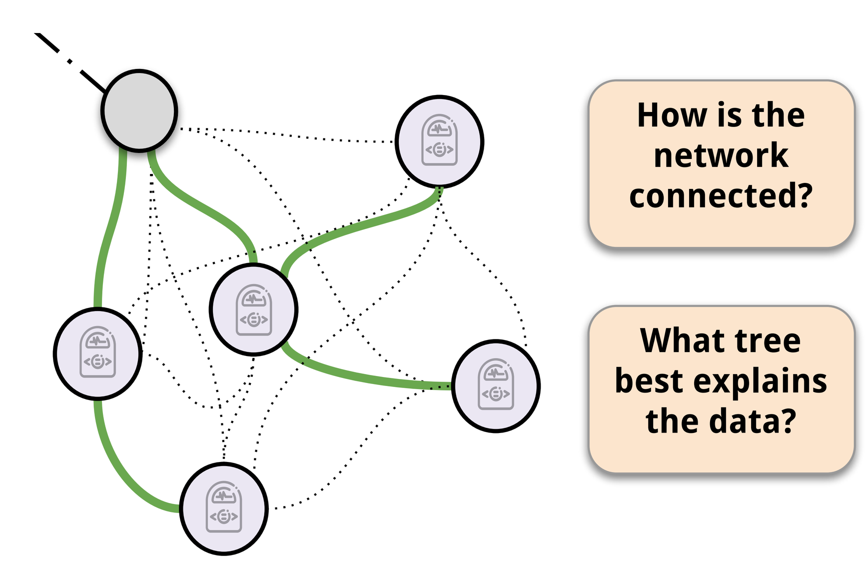

Figure 2. Topology learning problem: Ensuring grid model accuracy with smart meter measurements.

Suppose that the m smart meter measurements have quantization width $\Delta > 0$. Provided that

$$m \gtrsim 2n \log\left(\frac{n+1}{2}\right) + \frac{3}{2}n, \tag{5}$$

then the topology estimate from solving (4) with an appropriate choice of $\mathcal{K}$ obtains an error bounded, with exponentially high probability, as

$$\|\hat{\boldsymbol{x}} - \boldsymbol{x}_0\|_2 \lesssim \Delta \sqrt{\frac{n \log\left(\frac{n+1}{2}\right) + \frac{3}{2}n}{m}}. \tag{6}$$

Key idea: This error bound is better than what we could hope to achieve in general for a meshed system, because we are leveraging the sparse structure of $\boldsymbol{x}_0$ that arises in radial networks.

Numerical Results

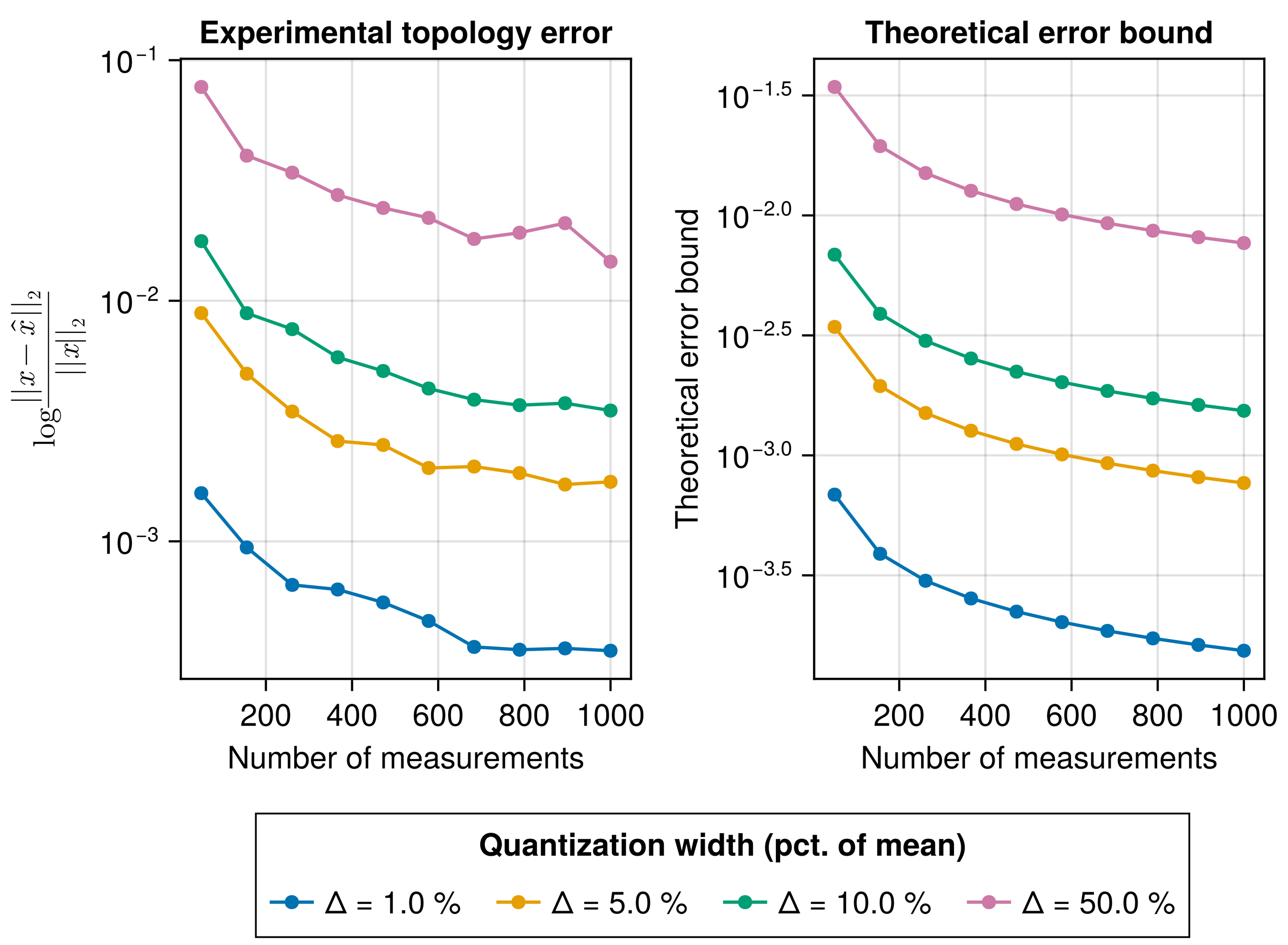

Figure 3. Relative error vs. number of measurements for learning the topology of the `case33bw` test network with the convex program (4), by quantization width. The leftmost pane is experimental results, while the rightmost pane shows the error prescribed by (6), which accurately predicts the performance of the algorithm.

Conclusion and Future Work

The key findings are an **efficient method** and **error bounds** for learning a distribution network topology. The method is robust to a generic class of non-Gaussian measurement noise, and supports **variable precision in the measurements**. By embedding the tree structure of distribution networks into the estimation problem, we show that the proposed method gives **precise predictions** of the grid topology. Non-asymptotic error bounds allow for the performance of the algorithm to be predicted without requiring an optimization problem to be solved.

References

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