Electricity Market Regulation via Differentiable Optimization

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How does network infrastructure impact energy affordability?

- Underlying network structure impacts electricity prices, described by the locational marginal price (LMP): the cost of serving an additional unit of load, constrained by network congestion and losses in the wholesale market.
- Commonly used metric of energy affordability: **energy burden**—a function of electricity rates, income, and electricity usage [1].
- We develop an analogous concept: **Locational marginal energy burden** (LMB) describes the change in energy burden incurred by serving one additional unit of demand at that bus [2].

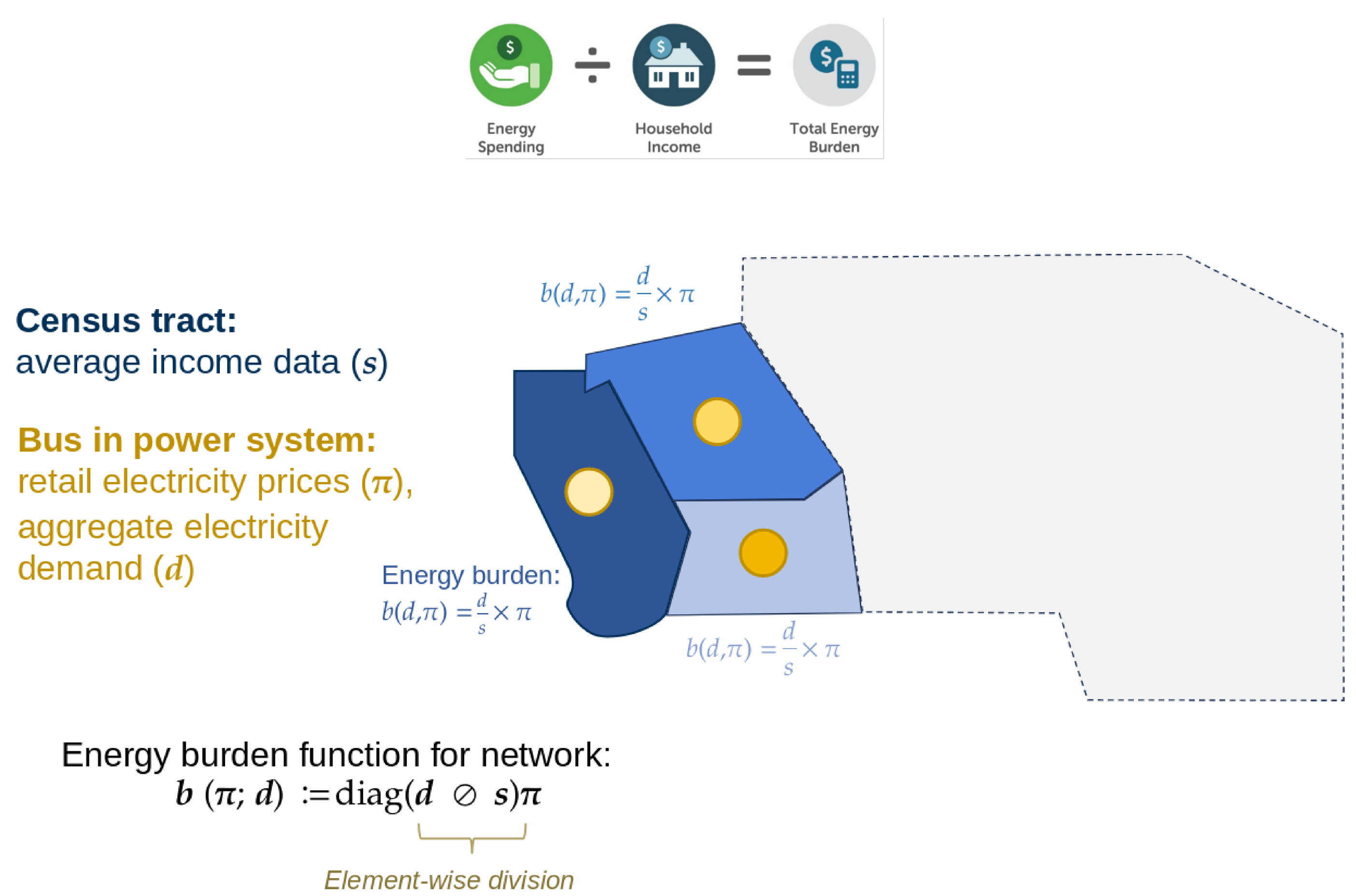

Figure 1. The energy burden metric: a classical measure of energy affordability.

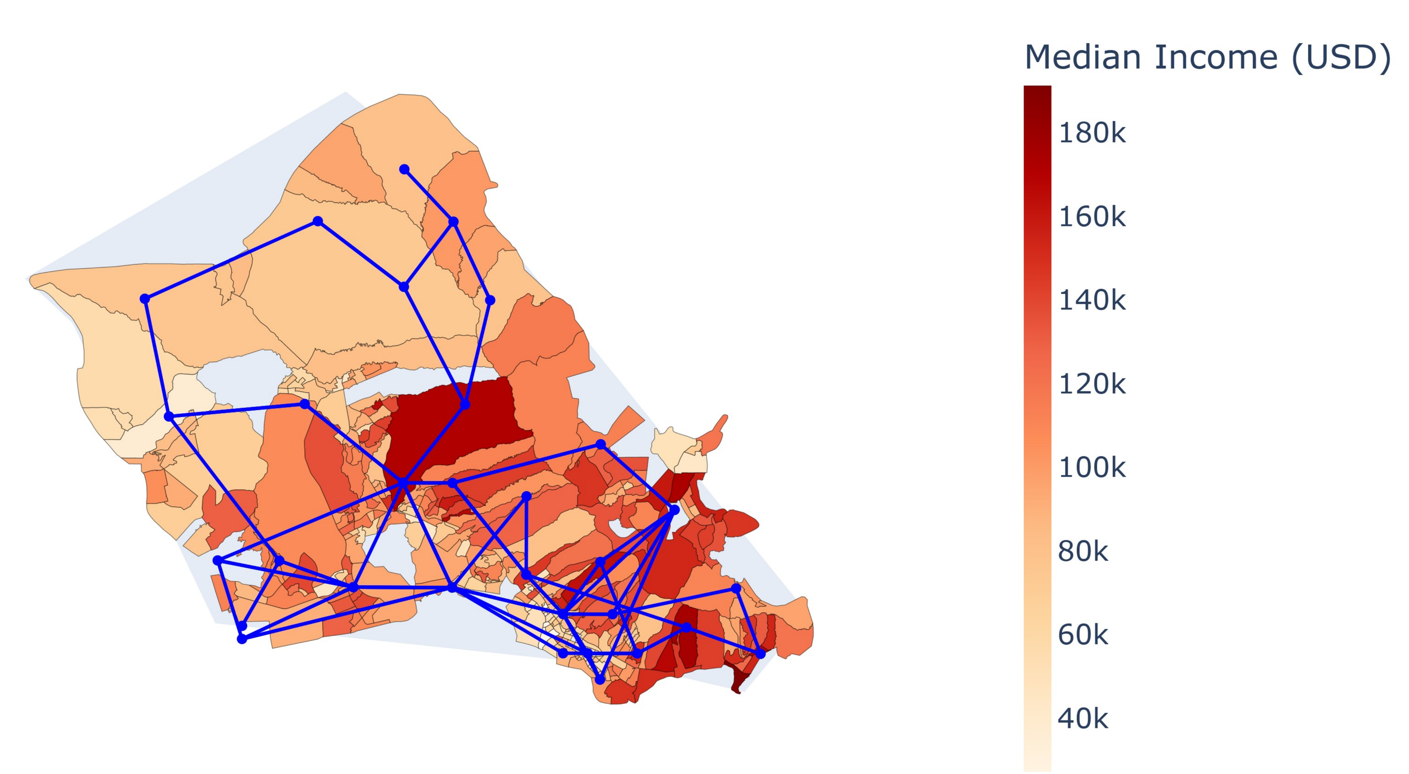

Figure 2. Mapping of publicly available household income data to the Hawaii test network [3].

Research approach

Parameterized OPF solution map: We consider a *parameterized solution map* of the (DC) OPF problem that returns operational decisions $\boldsymbol{x}^\star$ given chosen problem parameters $\boldsymbol{\eta}$:

$$\boldsymbol{x}^\star(\boldsymbol{\eta}) = \arg\min_{\boldsymbol{x} \in \mathcal{X}(\boldsymbol{\eta})} \quad c(\boldsymbol{x})$$

Retail pricing: The optimal dual variables of the power balance constraint $\boldsymbol{\nu}^\star(\boldsymbol{\eta})$ define *locational marginal prices* (LMPs) at each bus. The *retail price* $\boldsymbol{\pi}$ published by operators *is an implicit function of the LMPs*, and determines what price customers pay:

$$\text{retail-price} = \boldsymbol{\pi}\left(\boldsymbol{\nu}^\star(\boldsymbol{\eta})\right)$$

Implicit differentiation: The OPF solution can be differentiated with respect to problem parameters via the implicit function theorem [4, 5].

Locational marginal energy burden (LMB)

The energy burden function $\boldsymbol{b}$ provides a metric of energy affordability for all buses in the network, given customer incomes $\boldsymbol{s}$ and demands $\boldsymbol{d}$:

$$\boldsymbol{b} = \mathrm{diag}(\boldsymbol{d} \oslash \boldsymbol{s})\boldsymbol{\pi}. \tag{1}$$

The locational marginal energy burden (LMB) of a bus i induced by a bus j is the change in energy burden at bus i with respect to change in electricity demand at a bus j:

$$\frac{\partial b_i}{\partial d_j} = \frac{\text{change in energy burden at bus } i}{\text{change in demand at bus } j}. \tag{2}$$

The *LMB-to-others* induced by bus i is the net change in energy burden to *all other buses* $j \neq i$ with respect to change in electricity demand at bus i:

$$\text{LMB-to-others}(i) = \sum_{j: j \neq i} \frac{\partial b_j}{\partial d_i}. \tag{3}$$

Example: Energy burden regulation problem

Expansion planning task: Find a *grid infrastructure investment policy* $\boldsymbol{\eta} \in \mathcal{H}$ that reduces the energy burden of consumers below a threshold:

$$\min_{\boldsymbol{\eta} \in \mathcal{H}} \quad f\left(\boldsymbol{\nu}^\star(\boldsymbol{\eta})\right),$$

where $\boldsymbol{\nu}^\star$ is the dual optimal solution of lower-level OPF (i.e., LMPs).

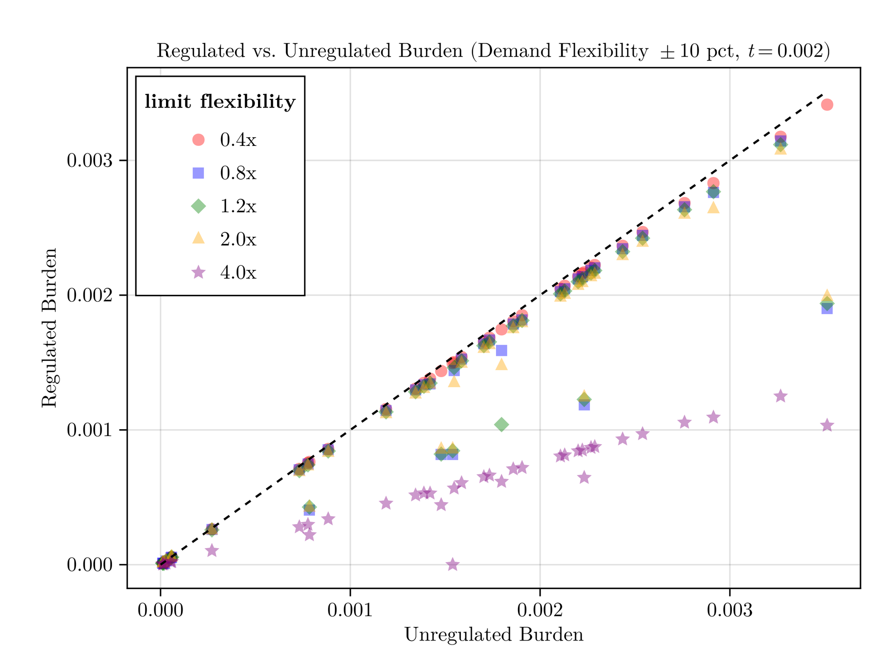

Figure 3. Regulating energy burden by imposing grid infrastructure investments.

Numerical results

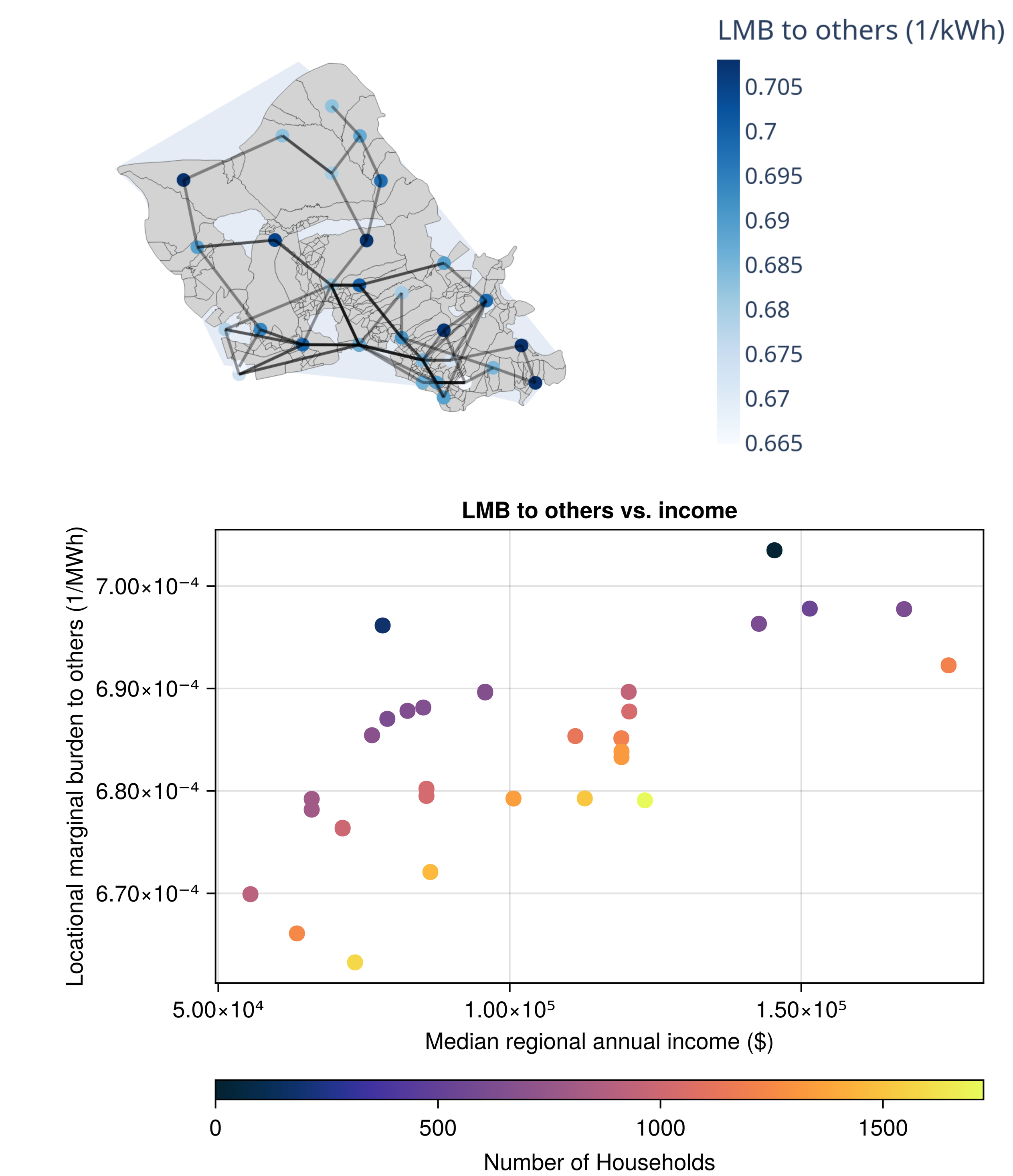

Figure 4. Energy consumption in high-income, low-population-density areas disproportionately impacts the energy burden faced by others.

Conclusion and future work

Differentiable optimization **operationalizes** energy equity metrics via locational marginal energy burden (LMB). By integrating the LMB metric into market clearing algorithms, we will develop an efficient **algorithmic policymaking tool** that enables the *regulation of fair pricing mechanisms in electricity markets* while simultaneously modeling operational grid constraints.

References

[1] E. Baker, S. Carley, S. Castellanos, D. Nock, J. F. Bozeman, D. Konisky, C. G. Monyei, M. Shah, and B. Sovacool, "Metrics for decision-making in energy justice," *Annual Review of Environment and Resources*, vol. 48, pp. 737–760, 2023.

[2] S. Talkington, A. West, and R. Haider, "Locational marginal burden: Quantifying the equity of optimal power flow solutions," in *Proceedings of the 15th ACM International Conference on Future and Sustainable Energy Systems*, e-Energy '24, (New York, NY, USA), p. 97–107, Association for Computing Machinery, 2024.

[3] A. B. Birchfield, T. Xu, K. M. Gegner, K. S. Shetye, and T. J. Overbye, "Grid Structural Characteristics as Validation Criteria for Synthetic Networks," *IEEE Transactions on Power Systems*, vol. 32, pp. 3258–3265, July 2017.

[4] A. Agrawal, B. Amos, S. Barratt, S. Boyd, S. Diamond, and Z. Kolter, "Differentiable convex optimization layers," 2019.

[5] L. F. Valenzuela, A. Degleris, A. E. Gamal, M. Pavone, and R. Rajagopal, "Dynamic locational marginal emissions via implicit differentiation," *IEEE Transactions on Power Systems*, pp. 1–11, 2023.